hep-ph/0102280

MRI-P-010204

INLO-PUB-01/2001

QCD and power corrections to sum rules

in deep-inelastic lepton-nucleon scattering

V. Ravindran
Harish-Chandra Research Institute,
Chhatnag Road, Jhusi, Allahabad-211019,
India.

W.L. van Neerven
Instituut-Lorentz,
University of Leiden,
PO Box 9506, 2300 RA Leiden,
The Netherlands.

February 2001

Abstract

In this paper we study QCD and power corrections to sum rules which show up in deep-inelastic lepton-hadron scattering. Furthermore we will make a distinction between fundamental sum rules which can be derived from quantum field theory and those which are of a phenomenological origin. Using current algebra techniques the fundamental sum rules can be expressed into expectation values of (partially) conserved (axial-) vector currents sandwiched between hadronic states. These expectation values yield the

quantum numbers of the corresponding hadron which are determined by the underlying flavour group $SU(n)_F$. In this case one can show that there exist an intimate relation between the appearance of power and QCD corrections. The above features do not hold for phenomenological sum rules, hereafter called non-fundamental. They have no foundation in quantum field theory and they mostly depend on certain assumptions made for the structure functions like superconvergence relations or the parton model. Therefore only the fundamental sum rules provide us with a stringent test of QCD.

PACS: 11.50.Li, 12.38.Bx, 13.60.Hb
Keywords: Sum rules, Perturbative calculations, Deep-inelastic processes.

1 Introduction

Sum rules in deep-inelastic lepton hadron scattering provide us with one of the most beautiful tools to test the predictions of perturbative QCD [1]. This is because they can be expressed into integrals of the type $\int_0^1 dx\ \Delta F^N(x,q^2) = A_N$ where $\Delta F^N(x,q^2)$ either denotes a structure function or a combination of structure functions and N represents the corresponding hadron in the initial state of the deep-inelastic process. In this way one gets rid of the unknown x-dependence which is due to non-perturbative effects. However this statement only holds if A_N can be determined in an unambiguous way as we will elucidate in this paper. Furthermore sum rules can be computed up to much higher orders in perturbation theory than other quantities which are mostly known up to next-to-leading order only. Examples are the Bjorken sum rules [2], [3] and the Gross- Llewellyn Smith sum rule [4] which have been calculated up to order α_s^3 in [5], [6]. The same features are shown by the total cross section in $e^+\ e^- \to hadrons$ [7] and the width of the Z-boson [8]. The only problem is that it is very hard to measure the sum rules experimentally since the structure functions are only known for a limited range of x. Therefore the uncertainties are due to extrapolations of the structure functions into the small and large x-region. Fortunately the small x-region is not so important since all sum rules only hold for the non-singlet parts of the structure functions which tend to zero when $x \to 0$. The large x-region is important and here the data are mainly coming from fixed target experiments. More information about this region will come after the upgrade of HERA [9]. Furthermore we place our hope on polarized electron-proton colliders [10], [11] which allows us to measure the parity violating structure functions $g_i(x,q^2)$ $(i = 3-5)$ for which some interesting sum rules can be derived. In the literature one can find many sum rules (see e.g. [12], [13], [14], [15]). However from a theoretical point of view they cannot be put on an equal footing. In this paper we make a clear distinction between fundamental and non-fundamental sum rules. In the former case the quantity A_N is given by the expectation value of a conserved current or partially conserved axial vector current sandwiched between the hadronic state N. Furthermore the (axial-) vector currents are put in the adjoint representation of the underlying flavour group given by $SU(n)_F$ where n denotes the number of light flavours. These sum rules can either be derived from equal time current algebra [16] or from light-cone current algebra [17] [18]. Examples are the sum rules given by Adler [19], by Bjorken [2] [3] and by Gross and Llewellyn Smith [4]. The Ellis-Jaffe sum rule [20] does not belong to this class since the singlet axial-vector current is not conserved due to the Adler-Bell-Jackiw anomaly [21]. Therefore it acquires an infinite renormalization which induces scaling violating terms in the perturbation series for A_N so that it becomes a non-fundamental sum rule. To the latter class also belong all sum rules which cannot be derived from current algebra. For instance the sum rules given by Gottfried [22], Burkhardt and Cottingham [23] cannot be expressed into expectation values of (axial-)vector current operators. Many more sum rules of this type can be found in [15]. In this paper we will give a complete list of fundamental sum rules using the techniques of current algebra. Further we investigate the perturbation

series which also includes a study of the power corrections. It turns out that there is an intimate relation between the appearance of non-zero higher order corrections and the presence of power corrections which can be either due to higher twist operators or mass dependent terms of the type m^2/q^2 which vanish in the limit $-q^2 \to \infty$. Here one has to bear in mind that sum rules do not acquire target mass correction since the spin of the operators (here (axial-) vector currents) is smaller than two. Apparently this also holds when the sum rule cannot be related to expectation values of (axial-) vector currents like the one given by Burkhardt and Cottingham [23] (see the treatment of target mass corrections in [24]). The paper is organized as follows. In section 2 we rederive the Adler sum rule for the unpolarized structure function $F_2(x, q^2)$ from the equal time current algebra using infinite frame techniques. One of the features is that this sum rule neither receives QCD corrections nor power contributions. It turns out that a similar sum rule exists for the polarized structure function $g_4(x, q^2)$ appearing in neutrino-nucleon scattering. In section 3 we show that the remaining sum rules can be obtained from light-cone current algebra. Since the region outside the light-cone is not taken into account these sum rules acquire power corrections and the perturbation series receives non-zero contributions beyond lowest order. In section 4 we study the power corrections by keeping a non-zero mass in the calculation of the first moment of the coefficient function. Here we show that when mass dependent terms appear one also encounters non-vanishing higher order corrections. At the end we discuss some peculiar sum rules for which $A_N = 0$ up to order α_s in perturbation theory. One of them is the Burkhardt Cottingham sum rule [23]. In the derivation we assume that the underlying flavour symmetry is given by the group $SU(3)$. The formulae for $SU(4)$ which are more appropriate at large $-q^2$ are presented in Appendix A. The long expressions for the partonic structure functions needed for the computation of the QCD corrections to the sum rules are given in Appendix B.

2 Infinite momentum frame techniques

In this chapter we use the infinite momentum frame technique to derive a sum rule for the longitudinally polarized structure function $g_4(x, q^2)$ which is the analogue of the Adler sum rule [19] in unpolarized (anti-) neutrino-nucleon scattering. Before the derivation we first give the definitions for the structure functions which emerge in deep-inelastic lepton-hadron scattering

$$l(k, \lambda) + N(p, s) \to l'(k') +' X' \,, \tag{2.1}$$

where l and l' denote the incoming and outgoing leptons and N represents the incoming hadron. The inclusive final hadronic state is given by $'X'$. Furthermore we have indicated between the brackets the momenta (k, k', p) and spins (λ, s) of the particles. In lowest order of the electroweak standard model the above reaction proceeds via the exchange of one of the vector bosons γ, Z (neutral current process) or $W^\pm$ (charged current process). Following the notations in [14], [15] the hadronic tensor $W^{\mu\nu}$ is defined by

$$W^{\mu\nu}_{(V_1V_2)}(p, q, s) \;=\; \frac{1}{4\pi}\int d^4z\, e^{i\,q\cdot z} \;< N(p, s)|\; [J^{\mu\;\dagger}_{V_1}(z), J^\nu_{V_2}(0)]\; |N(p, s) > \;. \tag{2.2}$$

Here V_1, V_2 refer to the intermediate vector bosons $V_i = \gamma, Z, W^\pm$. which appear in reaction (2.1). Furthermore we have $p^2 = m^2$, $s^2 = -1$ and $s \cdot p = 0$. In the case $V_1 \neq V_2$ we only consider the tensor $W^{\mu\nu}_{(V_1V_2)} + W^{\mu\nu}_{(V_2V_1)}$. Using Lorentz covariance and time-reversal invariance we can express the hadronic tensor in terms of fourteen structure functions

$$\begin{aligned}
W^{\mu\nu}_{(V_1V_2)}(p, q, s) \;=\; & -\tilde{g}^{\mu\nu} F_1^{V_1V_2}(x, q^2) + \frac{\tilde{p}^\mu \tilde{p}^\nu}{p\cdot q} F_2^{V_1V_2}(x, q^2) + i\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2\,p\cdot q} F_3^{V_1V_2}(x, q^2) \\
& + \frac{q^\mu q^\nu}{p\cdot q} F_4^{V_1V_2}(x, q^2) + \frac{p^\mu q^\nu + p^\nu q^\mu}{2\,p\cdot q} F_5^{V_1V_2}(x, q^2) \\
& + im\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha s_\beta}{p\cdot q} g_1^{V_1V_2}(x, q^2) + im\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha(p\cdot q s_\beta - s\cdot q p_\beta)}{(p\cdot q)^2} g_2^{V_1V_2}(x, q^2) \\
& + \frac{m}{p\cdot q}\left(\frac{\tilde{p}^\mu \tilde{s}^\nu + \tilde{p}^\nu \tilde{s}^\mu}{2} - s\cdot q\frac{\tilde{p}^\mu\tilde{p}^\nu}{(p\cdot q)}\right) g_3^{V_1V_2}(x, q^2) \\
& + ms\cdot q\frac{\tilde{p}^\mu\tilde{p}^\nu}{(p\cdot q)^2} g_4^{V_1V_2}(x, q^2) - m\tilde{g}^{\mu\nu}\frac{s\cdot q}{p\cdot q} g_5^{V_1V_2}(x, q^2) \\
& + i\; m\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha s_\beta}{p\cdot q} g_6^{V_1V_2}(x, q^2) + m\; s\cdot q\;\frac{q^\mu q^\nu}{(p\cdot q)^2} g_7^{V_1V_2}(x, q^2) \\
& + m\; s\cdot q\;\frac{p^\mu q^\nu + p^\nu q^\mu}{2(p\cdot q)^2} g_8^{V_1V_2}(x, q^2) + m\;\frac{s^\mu q^\nu + s^\nu q^\mu}{2p\cdot q} g_9^{V_1V_2}(x, q^2)\,,
\end{aligned} \tag{2.3}$$

with the following shorthand notations

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\,, \quad \tilde{p}^\mu = p^\mu - \frac{p\cdot q}{q^2} q^\mu\,, \quad \tilde{s}^\mu = s^\mu - \frac{s\cdot q}{q^2} q^\mu\,. \tag{2.4}$$

Instead of the set of structure functions above one can make other choices (see e.g. [25]) as long as all structure functions are linearly independent. The unpolarized $F_i^{V_1V_2}(x,q^2)$ $(i=1-5)$ and the polarized structure functions $g_i^{V_1V_2}(x,q^2)$ $(i=1-9)$ depend besides on the virtuality of the intermediate vector boson $-q^2>0$ and the Bjorken scaling variable $x=-q^2/2p\cdot q$ also on the mass of the hadron. Contraction of the hadronic tensor with the leptonic tensor provides us with the cross section which can be found in Eq. (11) of [15]. When the lepton masses are neglected the leptonic current is conserved so that the structure functions $F_4,\ F_5$ and $g_7,\ g_8,\ g_9$ drop out of the cross section. Notice that the structure function g_6 also contributes if $\vec{s}\parallel\vec{q}$ which is often ignored in the literature. The electroweak currents are in general decomposed into a vector current V_a^μ and an axial-vector current A_a^μ where a indicates that these currents belong to the adjoint representation of the flavour group $SU(n)_F$. Notice that the structure functions F_i $(i=1-5)$ and $g_1,\ g_2,\ g_6$ only get contributions from vector vector and axial-vector axial-vector correlation functions whereas $F_3, g_3,\ g_4,\ g_5$ and g_i $(i=7-9)$ are determined by vector-axial-vector combinations.

According to the hypothesis of Gell-Mann in [16], the zero components of the vector and axial-vector currents satisfy the following equal time commutation (ETC) algebra

$$\begin{aligned}\left[V_a^0(0,\vec{z}), V_b^0(0,\vec{y})\right] &= \left[A_a^0(0,\vec{z}), A_b^0(0,\vec{y})\right] = i\delta^{(3)}(\vec{z}-\vec{y})\, f_{abc} V_c^0(z)\,,\\ \left[V_a^0(0,\vec{z}), A_b^0(0,\vec{y})\right] &= \left[A_a^0(0,\vec{z}), V_b^0(0,\vec{y})\right] = i\delta^{(3)}(\vec{z}-\vec{y})\, f_{abc} A_c^0(z)\,.\end{aligned} \tag{2.5}$$

In the expressions above f_{abc} denote the structure constants of the Lie-algebra of $SU(n)_F$ which are defined by $[\lambda_a,\lambda_b]=2\,i\,f_{abc}\,\lambda_c$ where λ_a are the generators of the Lie-algebra. These relations are satisfied irrespective of the origin of the currents so that it does not matter whether they are composed of fermionic or bosonic fields. We can now compute the integral

$$\begin{aligned}\int_{-\infty}^{\infty} d\,q_0\, W_{ab,VV}^{00} &= \frac{1}{2}\int d^3\vec{z}\, e^{-i\vec{q}\cdot\vec{z}} \langle N(p,s)|[V_a^0(0,\vec{x}), V_b^0(0,\vec{0})]|N(p,s)\rangle =\\ &\frac{i}{2} f_{abc}\langle N(p,s)|V_c^0(0)|N(p,s)\rangle\,.\end{aligned} \tag{2.6}$$

Similar results are obtained for

$$\int_{-\infty}^{\infty} d\,q_0\, W_{ab,AA}^{0\nu} = \frac{i}{2} f_{abc}\langle N(p,s)|V_c^\nu(0)|N(p,s)\rangle\,, \tag{2.7}$$

$$\int_{-\infty}^{\infty} d\,q_0\, W_{ab,VA}^{0\nu} = \frac{i}{2} f_{abc}\langle N(p,s)|A_c^\nu(0)|N(p,s)\rangle\,, \tag{2.8}$$

$$\int_{-\infty}^{\infty} d\,q_0\, W_{ab,AV}^{0\nu} = \frac{i}{2} f_{abc}\langle N(p,s)|A_c^\nu(0)|N(p,s)\rangle\,. \tag{2.9}$$

The expectation values of the vector and axial-vector currents are given by

$$\langle N(p,s)|V_c^\nu(0)|N(p,s)\rangle = \Gamma_c^N\, p^\nu\,, \quad \langle N(p,s)|A_c^\nu(0)|N(p,s)\rangle = m\,\Gamma_c^{5,N}\, s^\nu\,. \tag{2.10}$$

In order to compute the sum rules we can choose an infinite momentum frame where the momenta have the following components

$$p = \left(P + \frac{m^2}{2\,P}, \vec{0}_\perp, P\right)\,, \quad q = \left(\frac{m\,\nu}{P}, \vec{q}_\perp, 0\right)\,. \tag{2.11}$$

so that $\nu = p{\cdot}q/m$ is satisfied. In this frame the longitudinal spin has large components only. Because $s\cdot p = 0$, the components of the longitudinal spin can be written as

$$s = \left(\frac{P}{m}, 0_\perp, \frac{P}{m} + \frac{m}{2P}\right)\,. \tag{2.12}$$

In the limit $P \to \infty$ we obtain

$$\int_{-\infty}^{\infty} d\,q_0\, W^{00}_{ab,VV} = \int_{-\infty}^{\infty} d\,q_0\, W^{00}_{ab,AA} = P\,\int_{-\infty}^{\infty} \frac{d\,\nu}{\nu}\, F_{2,ab,VV}(\nu,q^2)\,,$$

$$\int_{-\infty}^{\infty} d\,q_0\, W^{00}_{ab,AV} = \int_{-\infty}^{\infty} d\,q_0\, W^{00}_{ab,VA} = P\,\int_{-\infty}^{\infty} \frac{d\,\nu}{\nu}\, g_{4,ab,AV}(\nu,q^2)\,. \tag{2.13}$$

In this paper we will not discuss the justification of the infinite momentum frame technique which seems to us rather formal. For more details we refer to [12]. From Eqs. (2.6)-(2.10), (2.13) we obtain the following results.

$$\int_{-1}^{1} \frac{d\,x}{x}\, F_{2,ab,VV}(x,q^2) = \int_{-\infty}^{\infty} \frac{d\,\nu}{\nu}\, F_{2,ab,VV}(\nu,q^2) = \frac{i}{2} f_{abc}\Gamma_c^N\,,$$

$$\int_{-1}^{1} \frac{d\,x}{x}\, g_{4,ab,AV}(x,q^2) = \int_{-\infty}^{\infty} \frac{d\,\nu}{\nu}\, g_{4,ab,AV}(\nu,q^2) = \frac{i}{2} f_{abc}\Gamma_c^{5,N}\,. \tag{2.14}$$

The formulae above still do not represent the sum rules since they have to be converted into integrals over the physical region $0 < x < 1$. Since the ETC algebra in Eq. (2.5) only involves the Lie-algebra structure constants f_{abc} the sum rules can be only derived for charged current processes. Choosing three flavours i.e. $n = 3$ the charged currents admit the following representation

$$J_\pm^\mu(y) = \Big(V^\mu_{1\pm i2}(y) - A^\mu_{1\pm i2}(y)\Big)\cos\theta_c + \Big(V^\mu_{4\pm i5}(y) - A^\mu_{4\pm i5}(y)\Big)\sin\theta_c\,, \tag{2.15}$$

where θ_c denotes the Cabibbo angle. Further we have introduced the following short-hand notations

$$J_+^\mu \equiv J^\mu_{W^+}\,, \quad J_-^\mu \equiv J^\mu_{W^-}\,, \quad J^\mu_{a\pm ib} \equiv J_a^\mu \pm i\,J_b^\mu\,, \tag{2.16}$$

so that the corresponding structure tensor reads

$$W_\pm^{\mu\nu} \;=\; \frac{1}{4\pi}\int d^4z\, e^{i\,q\cdot z}\,\langle N(p,s)|\;[J_\pm^{\mu\;\dagger}(z), J_\pm^\nu(0)]\;|N(p,s)\rangle\,. \tag{2.17}$$

Further we infer from Eq. (2.15) the property $J_\pm^{\mu\,\dagger} = J_\mp^\mu$. Notice that the current J_+^μ appears in the process

$$\nu + N \to l^- +' X' \,, \quad \text{or} \quad l^+ + N \to \bar{\nu} +' X' \,, \tag{2.18}$$

which involves the W^+ exchange whereas J_-^μ shows up in

$$\bar{\nu} + N \to l^+ +' X' \,, \quad \text{or} \quad l^- + N \to \nu +' X' \,, \tag{2.19}$$

which proceeds via W^- exchange. Substitution of the charged current Eq. (2.15) into the commutator of currents Eq. (2.17) leads to the result

$$\begin{aligned} W_\pm^{\mu\nu} &= \pm 4\, i\, \cos^2\theta_c (W_{12,VV}^{\mu\nu} - W_{12,AV}^{\mu\nu}) \pm 4\, i\, \sin^2\theta_c (W_{45,VV}^{\mu\nu} - W_{45,AV}^{\mu\nu}) \\ &\quad \mp 4\, i\, \sin\theta_c\, \cos\theta_c (W_{24,VV}^{\mu\nu} - W_{15,VV}^{\mu\nu} - W_{24,AV}^{\mu\nu} + W_{15,AV}^{\mu\nu}) \,. \end{aligned} \tag{2.20}$$

Since the AA-part is equal to the VV-part (see Eq. (2.5)) we do not distinguish them anymore. The same also holds for the VA-part and the AV-part. From translation invariance (see Eq. (3.8)) and $J_\pm^{\mu\,\dagger} = J_\mp^\mu$ one can derive

$$\langle N(p,s)|\, [J_\pm^{\mu\,\dagger}(z), J_\pm^\nu(0)]\, |N(p,s)\rangle = -\langle N(p,s)|\, [J_\mp^{\mu\,\dagger}(-z), J_\mp^\nu(0)]\, |N(p,s)\rangle \,, \tag{2.21}$$

Substitution of the equation above into Eq. (2.2) provides us with the relation

$$W_\pm^{\mu\nu}(p,q,s) = -W_\mp^{\mu\nu}(p,-q,s) \,. \tag{2.22}$$

Hence we have

$$F_2^{\mathrm{W^-N}}(x,q^2) = F_2^{\mathrm{W^+N}}(-x,q^2) \,, \quad g_4^{\mathrm{W^-N}}(x,q^2) = g_4^{\mathrm{W^+N}}(-x,q^2) \,. \tag{2.23}$$

We can now derive the following sum rules for three flavours. From the VV-part we obtain the Adler sum rule

$$\begin{aligned} \int_{-1}^1 \frac{d\,x}{x}\, F_2^{\mathrm{W^-N}}(x,q^2) &= \int_0^1 \frac{d\,x}{x} \Big(F_2^{\mathrm{W^-N}}(x,q^2) - F_2^{\mathrm{W^+N}}(x,q^2) \Big) = \\ & (2\, I_3^N + 3\, Y^N) + \cos^2\theta_c\, (2\, I_3^N - 3\, Y^N) \,, \end{aligned} \tag{2.24}$$

where the vector charges (isospin, hypercharge and baryon number) are given by

$$\Gamma_3^N = 2\, I_3^N \,, \quad \Gamma_8^N = \sqrt{3}\, Y^N \,, \quad \Gamma_0^N = 3\, B^N \,, \quad \Gamma_6^N = 0 \,, \tag{2.25}$$

with $Y = S + B$ where S denotes the strangeness of the hadron N.
In the case of polarized scattering we have the following analogue for the Adler sum rule which follows from the VA-part.

$$\begin{aligned} \int_{-1}^1 \frac{d\,x}{x} g_4^{\mathrm{W^-N}}(x,q^2) &= \int_0^1 \frac{d\,x}{x} \Big(g_4^{\mathrm{W^-N}}(x,q^2) - g_4^{\mathrm{W^+N}}(x,q^2) \Big) = \\ \Big\{ -2\, I_3^N(F+D) - (3\,F - D) \Big\} &+ \cos^2\theta_c \Big\{ -2\, I_3^N(F+D) + (3\,F - D) \Big\} \,. \end{aligned} \tag{2.26}$$

For the axial vector charges we have

$$\Gamma_3^{5,N} = 2\, I_3^N (F + D)\,, \quad \Gamma_8^{5,N} = \frac{1}{\sqrt{3}}\,(3\,F - D)\,, \quad \Gamma_6^{5,N} = 0\,, \tag{2.27}$$

with $g_A = F + D = 1.254 \pm 0.006$ and $3\,F - D = 0.68 \pm 0.04$. These values follow from the fact that the hadronic axial-vector current is only partially conserved i.e. $\partial_\mu A_c^\mu \neq 0$. In the case of the proton (p) and the neutron (n) the quantum numbers above are given by

$$I_3^p = \frac{1}{2}\,, \quad B^p = 1\,, \quad S^p = 0\,,$$

$$I_3^n = -\frac{1}{2}\,, \quad B^n = 1\,, \quad S^n = 0\,. \tag{2.28}$$

The most important feature of the two sum rules above is that they are exact and therefore model independent contrary to the sum rules derived in the next section. This means that they also hold beyond QCD. For instance the ETC relations in Eq. (2.5) also holds for bosonic currents. The reason is that at the tip of the light cone $z - y = 0$ all ETC relations take the same form irrespective of the origin (bosonic or fermionic) of the currents. Therefore in QCD the two sum rules have to be obeyed which implies that they do not receive any power corrections of the type $(1/q^2)^p$ which e.g. can be attributed to mass corrections or to higher twist contributions.

3 Light cone current algebra

In this section we derive the sum rules which hold when the currents are expressed into fermionic fields. This happens in QCD where the fermions are represented by the quarks. According to the hypothesis made in [17], [18] (see also [26]) the (axial-) vector currents satisfy the following light-cone algebra

$$\begin{aligned}
[V_a^\mu(z), V_b^\nu(y)] \underset{(z-y)^2\to 0}{=} & [A_a^\mu(z), A_b^\nu(y)] \underset{(z-y)^2\to 0}{=} -\frac{1}{2}\partial_\lambda \Delta(z-y) \\
& \Big[i\, s^{\mu\nu\lambda\sigma} f_{abc} \{V_{\sigma c}(z,y) + V_{\sigma c}(y,z)\} \\
& + s^{\mu\nu\lambda\sigma} d_{abc} \{V_{\sigma c}(z,y) - V_{\sigma c}(y,z)\} \\
& - \epsilon^{\mu\nu\lambda\sigma} f_{abc} \{A_{\sigma c}(z,y) - A_{\sigma c}(y,z)\} \\
& + i\, \epsilon^{\mu\nu\lambda\sigma} d_{abc} \{A_{\sigma c}(z,y) + A_{\sigma c}(y,z)\}\Big] \,, \qquad (3.1)
\end{aligned}$$

$$\begin{aligned}
[A_a^\mu(z), V_b^\nu(y)] \underset{(z-y)^2\to 0}{=} & [V_a^\mu(z), A_b^\nu(y)] \underset{(z-y)^2\to 0}{=} -\frac{1}{2}\partial_\lambda \Delta(z-y) \\
& \Big[i\, s^{\mu\nu\lambda\sigma} f_{abc} \{A_{\sigma c}(z,y) + A_{\sigma c}(y,z)\} \\
& + s^{\mu\nu\lambda\sigma} d_{abc} \{A_{\sigma c}(z,y) - A_{\sigma c}(y,z)\} \\
& - \epsilon^{\mu\nu\lambda\sigma} f_{abc} \{V_{\sigma c}(z,y) - V_{\sigma c}(y,z)\} \\
& + i\, \epsilon^{\mu\nu\lambda\sigma} d_{abc} \{V_{\sigma c}(z,y) + V_{\sigma c}(y,z)\}\Big] \,. \qquad (3.2)
\end{aligned}$$

Here $V_{\sigma c}(z,y)$ and $A_{\sigma c}(z,y)$ represent the bilocal vector and axial-vector currents respectively. Furthermore one has only kept the most singular pieces on the right-hand side which contribute to the light cone behaviour of the commutators. Therefore subleading terms, originating from mass corrections or higher twist contributions, are neglected. Another feature is the appearance of the symmetric structure constants d_{abc} which are defined by $\{\lambda_a, \lambda_b\} = 2\, d_{abc}\, \lambda_c$ where $d_{ab0} = \frac{2}{n}\, \delta_{ab}$ (see below Eq. (2.5)). The causal function $\Delta(z-y)$ appearing in the expressions above is given by

$$i\, \Delta(z-y) = \frac{1}{(2\pi)^3} \int d^4p\, \epsilon(p^0)\, \delta(p^2)\, e^{-ip\cdot(z-y)} \,, \qquad (3.3)$$

which has the properties

$$\Delta(z-y) = 0 \quad \text{for} \quad (z-y)^2 < 0 \,, \quad \partial_\lambda^z \Delta(z-y) = -g_{\lambda 0}\delta^{(3)}(\vec{z}-\vec{y}) \,. \qquad (3.4)$$

The current commutator algebra in Eqs. (3.1), (3.2) is satisfied by the free field bilocal currents

$$V_a^\mu(z,y) = \bar\psi(z)\gamma^\mu \frac{\lambda_a}{2}\psi(y)\,, \quad A_a^\mu(z,y) = \bar\psi(z)\gamma^\mu\gamma^5 \frac{\lambda_a}{2}\psi(y)\,. \tag{3.5}$$

Hence the local hermitian currents are obtained from the bilocal ones via the relations

$$V_a^\mu(z) \equiv V_a^\mu(z,z)\,, \qquad\qquad A_a^\mu(z) \equiv A_a^\mu(z,z)\,, \tag{3.6}$$

where $\psi(z)$ represents the mass-less free Dirac field. From the current algebra in Eqs. (3.1), (3.2) one can express the structure functions in Eq. (2.3) into Fourier transforms of the bilocal currents for $(z-y)^2 = 0$ provided one takes the Bjorken limit given by $-q^2 \to \infty$ with $x = -q^2/2p\cdot q$ is fixed. The procedure starts by sandwiching the commutators between the hadronic state $|N(p,s)\rangle$ so that the expectation values of the bilocal currents can be written as

$$\begin{aligned}
\langle N(p,s) \mid V_c^\sigma(z,0) + V_c^\sigma(0,z) \mid N(p,s)\rangle &= 2\,p^\sigma\, V_c^1(z^2, z\cdot p) + 2\,i\,z^\sigma\, V_c^2(z^2,z\cdot p)\,,\\
\langle N(p,s) \mid V_c^\sigma(z,0) - V_c^\sigma(0,z) \mid N(p,s)\rangle &= 2\,p^\sigma\, \bar V_c^1(z^2, z\cdot p) + 2\,i\,z^\sigma\, \bar V_c^2(z^2,z\cdot p)\,,\\
\langle N(p,s) \mid A_c^\sigma(z,0) + A_c^\sigma(0,z) \mid N(p,s)\rangle &= 2\,s^\sigma\, m\, A_c^1(z^2, z\cdot p)\\
&\quad -2\,i\,p^\sigma\; z\cdot s\, m\, A_c^2(z^2, z\cdot p)\\
&\quad +2\,z^\sigma\; z\cdot s\, m\, A_c^3(z^2, z\cdot p)\,,\\
\langle N(p,s) \mid A_c^\sigma(z,0) - A_c^\sigma(0,z) \mid N(p,s)\rangle &= 2\,s^\sigma\, m\, \bar A_c^1(z^2, z\cdot p)\\
&\quad -2\,i\,p^\sigma\; z\cdot s\, m\, \bar A_c^2(z^2, z\cdot p)\\
&\quad +2\,z^\sigma\; z\cdot s\, M\, \bar A_c^3(z^2, z\cdot p)\,.
\end{aligned} \tag{3.7}$$

Under translation invariance the bilocal currents transform like

$$e^{-i\,\hat P\cdot z}\, J_c^\mu(0,z)\, e^{i\,\hat P\cdot z} = J_c^\mu(-z,0)\,, \qquad J = V, A\,, \tag{3.8}$$

from which one can derive the following relations

$$\begin{aligned}
V_c^1(z^2, -z\cdot p) &= V_c^1(z^2, z\cdot p)\,, \quad V_c^2(z^2,-z\cdot p) = -V_c^2(z^2,z\cdot p)\,,\\
\bar V_c^1(z^2, -z\cdot p) &= -\bar V_c^1(z^2, z\cdot p)\,, \quad \bar V_c^2(z^2,-z\cdot p) = \bar V_c^2(z^2,z\cdot p)\,,\\
A_c^k(z^2, -z\cdot p) &= A_c^k(z^2, z\cdot p)\,, \quad k=1,3 \qquad A_c^2(z^2,-z\cdot p) = -A_c^2(z^2,z\cdot p)\,,\\
\bar A_c^k(z^2, -z\cdot p) &= -\bar A_c^k(z^2, z\cdot p)\,, \quad k=1,3 \qquad \bar A_c^2(z^2,-z\cdot p) = \bar A_c^2(z^2,z\cdot p)\,,
\end{aligned} \tag{3.9}$$

After insertion of the commutator algebra Eqs. (3.1), (3.2) into expression (2.2) one can compute the structure tensors which satisfy the relations $W_{ab}^{\mu\nu,VV} = W_{ab}^{\mu\nu,AA}$ and $W_{ab}^{\mu\nu,AV} = W_{ab}^{\mu\nu,VA}$. Here the superscripts VV, AA, AV, VA indicate the type of currents which appear in the commutators above. A straightforward calculation provides us with the following structure functions

$$
\begin{aligned}
2\,x\,F_{1,ab}^{VV}(x,q^2) &= F_{2,ab}^{VV}(x,q^2) = \frac{x}{2}\Big(i\,f_{abc}\,V_c^1(x) + d_{abc}\,\bar{V}_c^1(x)\Big)\,, \\
F_{3,ab}^{AV}(x,q^2) &= -\frac{1}{2}\Big(i\,f_{abc}\,\bar{V}_c^1(x) + d_{abc}\,V_c^1(x)\Big)\,, \\
F_{4,ab}^{VV}(x,q^2) &= F_{5,ab}^{VV}(x,q^2) = 0\,, \\
g_{1,ab}^{VV}(x,q^2) &= \frac{i}{4}\,f_{abc}\Big(\bar{A}_c^1(x) + \frac{\partial \bar{A}_c^2(x)}{\partial x}\Big) + \frac{1}{4}\,d_{abc}\Big(A_c^1(x) + \frac{\partial A_c^2(x)}{\partial x}\Big)\,, \\
g_{2,ab}^{VV}(x,q^2) &= -\frac{i}{4}\,f_{abc}\frac{\partial \bar{A}_c^2(x)}{\partial x} - \frac{1}{4}\,d_{abc}\frac{\partial A_c^2(x)}{\partial x}\,, \\
g_{3,ab}^{AV}(x,q^2) &= \frac{i}{2}\;f_{abc}\Big(x\,A_c^1(x) - A_c^2(x)\Big) + \frac{1}{2}\;d_{abc}\Big(x\,\bar{A}_c^1(x) - \bar{A}_c^2(x)\Big)\,, \\
2\,x\,g_{5,ab}^{AV}(x,q^2) &= g_{4,ab}^{AV}(x,q^2) = \\
&\quad i\,\frac{x}{2}\;f_{abc}\Big(A_c^1(x) + \frac{\partial A_c^2(x)}{\partial x}\Big) + \frac{x}{2}\;d_{abc}\Big(\bar{A}_c^1(x) + \frac{\partial \bar{A}_c^2(x)}{\partial x}\Big)\,, \\
g_{6,ab}^{VV}(x,q^2) &= -\frac{i}{4}\;f_{abc}\Big(x\,\bar{A}_c^1(x) - \bar{A}_c^2(x)\Big) - \frac{1}{4}\;d_{abc}\Big(x\,A_c^1(x) - A_c^2(x)\Big)\,, \\
g_{7,ab}^{AV}(x,q^2) &= 0\,, \\
x\,g_{9,ab}^{AV}(x,q^2) &= -x\,g_{8,ab}^{AV}(x,q^2) = \\
&\quad \frac{i}{4}\,f_{abc}\Big(x\,A_c^1(x) + A_c^2(x)\Big) + \frac{1}{4}\,d_{abc}\Big(x\,\bar{A}_c^1(x) + \bar{A}_c^2(x)\Big)\,, \qquad (3.10)
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
J_c^k(x) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} dz\cdot p\; e^{-i\,x\,p\cdot z}\, J_c^k(0,p\cdot z)\,, \quad x = -\frac{q^2}{2p\cdot q}\,, \\
J_c^k(0,p\cdot z) &= \int_{-1}^{1} dx\; e^{i\,x\,p\cdot z}\, J_c^k(x)\,, \quad J = V,\bar{V},A,\bar{A}\,. \qquad (3.11)
\end{aligned}
$$

Because of Eqs. (3.1), (3.2) one can identify $F_{i,ab}^{VV} = F_{i,ab}^{AA}$ and $F_{i,ab}^{AV} = F_{i,ab}^{VA}$. The same relations hold for the polarized structure functions $g_{i,ab}$. As we will see later

on these relations are also preserved in lowest order perturbation theory except for g_6 which turns out to depend on the mass assignment of the quarks in the Born reaction. In the above relations we have neglected all sub-leading terms which vanish in the Bjorken limit $-q^2 \to \infty$ like $1/q^2$. In this limit $F_4,\ F_5,\ g_7$ vanish and the matrix elements $V_c^2,\ \bar{V}_c^2,\ A_c^3,\ \bar{A}_c^3$ do not appear in the structure functions. Another feature is that the currents are composed of fermionic (spin half) fields (see Eq. (3.5)) which leads to the Callan-Gross relations $2xF_1 = F_2$ [27] and $2xg_5 = g_4$ [28] in Eq. (3.10). Furthermore one infers from Eq. (3.10) that the unpolarized structure functions F_i $(i = 1, 2, 4, 5)$ and the polarized structure functions $g_1,\ g_2,\ g_6$ only receive contributions from the commutators $[V, V]$ and $[A, A]$ (parity conserving) whereas the quantities F_3 and g_i $(i = 3, 4, 5, 7, 8, 9)$ are determined by the commutators $[A, V]$ and $[V, A]$ (parity violating) only. Further in the Bjorken limit only (leading) twist two contributions survive in the expressions for the structure functions F_i $(i = 1-3)$ and g_i $(i = 1, 4, 5)$ whereas the structure functions $g_2,\ g_3,\ g_6$. also receive twist three contributions. For an analysis see e.g. [14], [15]. We did a similar analysis for $g_8,\ g_9$ and found that these quantities are of twist three only. Notice that the latter structure functions do not show up in the cross section (see below Eq. (2.4)) when the masses of the leptons can be neglected. Hence from Eq. (3.10) one can conclude that the following matrix elements are of twist two

$$
\begin{aligned}
& V_c^1(x)\,, \qquad A_c^1(x) + \frac{\partial A_c^2(x)}{\partial x}\,, \\
& \bar{V}_c^1(x)\,, \qquad \bar{A}_c^1(x) + \frac{\partial \bar{A}_c^2(x)}{\partial x}\,.
\end{aligned} \tag{3.12}
$$

For the moment we only limit ourselves to the structure functions which receive contributions of twist two only and postpone the discussion of the other ones to the end of this section. From Eq. (3.9) one can derive the properties

$$
\begin{aligned}
V_c^1(-x) &= V_c^1(x)\,, \qquad V_c^2(-x) = -V_c^2(x)\,, \\
\bar{V}_c^1(-x) &= -\bar{V}_c^1(x)\,, \qquad \bar{V}_c^2(-x) = \bar{V}_c^2(x)\,, \\
A_c^k(-x) &= A_c^k(x)\,, \quad k = 1, 3\,, \qquad A_c^2(-x) = -A_c^2(x)\,, \\
\bar{A}_c^k(-x) &= -\bar{A}_c^k(x)\,, \quad k = 1, 3\,, \qquad \bar{A}_c^2(z^2, -x) = \bar{A}_c^2(x)\,.
\end{aligned} \tag{3.13}
$$

Like in the previous section we choose three flavours for our computations so that one gets the representation for the charged current in Eq. (2.15). For this choice the structure tensor in Eq. (2.17) becomes equal to

$$
\begin{aligned}
W_\pm^{\mu\nu} &= 2\cos^2\theta_c\Big(W_{11,VV}^{\mu\nu} + W_{22,VV}^{\mu\nu} + 2\,i\,W_{[12],VV}^{\mu\nu} - W_{11,AV}^{\mu\nu} - W_{22,AV}^{\mu\nu} \\
&\quad \mp 2\,i\,W_{[12],AV}^{\mu\nu}\Big) + 2\sin^2\theta_c\Big(W_{44,VV}^{\mu\nu} + W_{55,VV}^{\mu\nu} \pm 2\,i\,W_{[45],VV}^{\mu\nu} - W_{44,AV}^{\mu\nu}
\end{aligned}
$$

$$-W^{\mu\nu}_{55,AV} \mp 2\,i\,W^{\mu\nu}_{[45],AV}\Big) + 4\,\sin\theta_c\,\cos\theta_c (W^{\mu\nu}_{\{14\},VV} + W^{\mu\nu}_{\{25\},VV} \mp i\,W^{\mu\nu}_{[24],VV}$$

$$\pm i\,W^{\mu\nu}_{[15],VV} - W^{\mu\nu}_{\{14\},AV} - W^{\mu\nu}_{\{25\},AV} \pm i\,W^{\mu\nu}_{[24],AV} \mp i\,W^{\mu\nu}_{[15],AV}\Big)\,, \tag{3.14}$$

with the definitions

$$\begin{aligned} W^{\mu\nu}_{\{ab\},VV} &= \frac{1}{2}\Big(W^{\mu\nu}_{ab,VV} + W^{\mu\nu}_{ba,VV}\Big)\,, \\ W^{\mu\nu}_{[ab],VV} &= \frac{1}{2}\Big(W^{\mu\nu}_{ab,VV} - W^{\mu\nu}_{ba,VV}\Big)\,. \end{aligned} \tag{3.15}$$

From the equations above one can derive the charged current structure functions which are equal to

$$\begin{aligned} F_2^{\mathrm{W^+N}}(x)/x &= \cos^2\theta_c\Big[\frac{4}{3}\,\bar V_0^1(x) + \frac{2}{3}\sqrt{3}\,\bar V_8^1(x) - 2\,V_3^1(x)\Big] + \sin^2\theta_c\Big[\frac{4}{3}\,\bar V_0^1(x) \\ &\quad + \bar V_3^1(x) - \frac{1}{3}\sqrt{3}\,\bar V_8^1(x) - V_3^1(x) - \sqrt{3}\,V_8^1(x)\Big] \\ &\quad + 2\,\sin\theta_c\,\cos\theta_c\,\Big[\bar V_6^1(x) + V_6^1(x)\Big]\,. \end{aligned} \tag{3.16}$$

The structure function $F_2^{\mathrm{W^-N}}$ is obtained from $F_2^{\mathrm{W^+N}}$ by $V_a^1 \to -V_a^1$.

$$\begin{aligned} F_3^{\mathrm{W^+N}}(x) &= \cos^2\theta_c\Big[\frac{4}{3}\,V_0^1(x) + \frac{2}{3}\sqrt{3}\,V_8^1(x) - 2\,\bar V_3^1(x)\Big] + \sin^2\theta_c\Big[\frac{4}{3}\,V_0^1(x) \\ &\quad + V_3^1(x) - \frac{1}{3}\sqrt{3}\,V_8^1(x) - \bar V_3^1(x) - \sqrt{3}\,\bar V_8^1(x)\Big] \\ &\quad + 2\,\sin\theta_c\,\cos\theta_c\,\Big[V_6^1(x) + \bar V_6^1(x)\Big]\,. \end{aligned} \tag{3.17}$$

The structure function $F_3^{\mathrm{W^-N}}$ is obtained from $F_3^{\mathrm{W^+N}}$ by $\bar V_a^1 \to -\bar V_a^1$.

$$\begin{aligned} g_1^{\mathrm{W^+N}}(x) &= \cos^2\theta_c\Big[\frac{2}{3}\,\left(A_0^1(x) + \frac{\partial A_0^2(x)}{\partial x}\right) + \frac{1}{3}\sqrt{3}\,\left(A_8^1(x) + \frac{\partial A_8^2(x)}{\partial x}\right) \\ &\quad - \left(\bar A_3^1(x) + \frac{\partial \bar A_3^2(x)}{\partial x}\right)\Big] + \sin^2\theta_c\Big[\frac{2}{3}\,\left(A_0^1(x) + \frac{\partial A_0^2(x)}{\partial x}\right) \\ &\quad + \frac{1}{2}\left(A_3^1(x) + \frac{\partial A_3^2(x)}{\partial x}\right) - \frac{1}{6}\sqrt{3}\,\left(A_8^1(x) + \frac{\partial A_8^2(x)}{\partial x}\right) \\ &\quad - \frac{1}{2}\,\left(\bar A_3^1(x) + \frac{\partial \bar A_3^2(x)}{\partial x}\right) - \frac{1}{2}\sqrt{3}\,\left(\bar A_8^1(x) + \frac{\partial \bar A_8^2(x)}{\partial x}\right)\Big] \\ &\quad + \sin\theta_c\,\cos\theta_c\,\Big[\left(A_6^1(x) + \frac{\partial A_6^2(x)}{\partial x}\right) + \left(\bar A_6^1(x) + \frac{\partial \bar A_6^2(x)}{\partial x}\right)\Big]\,. \end{aligned} \tag{3.18}$$

The structure function $g_1^{\mathrm{W^-N}}$ is obtained from $g_1^{\mathrm{W^+N}}$ by $\bar{A}_a^i \to -\bar{A}_a^i$ $(i=1,2)$.

$$
\begin{aligned}
g_4^{\mathrm{W^+N}}(x)/x &= \cos^2\theta_c\Big[-\frac{4}{3}\left(\bar{A}_0^1(x)+\frac{\partial \bar{A}_0^2(x)}{\partial x}\right)-\frac{2}{3}\sqrt{3}\left(\bar{A}_8^1(x)+\frac{\partial \bar{A}_8^2(x)}{\partial x}\right) \\
&\quad +2\left(A_3^1(x)+\frac{\partial A_3^2(x)}{\partial x}\right)\Big]+\sin^2\theta_c\Big[-\frac{4}{3}\left(\bar{A}_0^1(x)+\frac{\partial \bar{A}_0^2(x)}{\partial x}\right) \\
&\quad -\left(\bar{A}_3^1(x)+\frac{\partial \bar{A}_3^2(x)}{\partial x}\right)+\frac{1}{3}\sqrt{3}\left(\bar{A}_8^1(x)+\frac{\partial \bar{A}_8^2(x)}{\partial x}\right) \\
&\quad +\left(A_3^1(x)+\frac{\partial A_3^2(x)}{\partial x}\right)+\sqrt{3}\left(A_8^1(x)+\frac{\partial A_8^2(x)}{\partial x}\right)\Big] \\
&\quad -2\sin\theta_c\cos\theta_c\Big[\left(\bar{A}_6^1(x)+\frac{\partial \bar{A}_6^2(x)}{\partial x}\right)+\left(A_6^1(x)+\frac{\partial A_6^2(x)}{\partial x}\right)\Big]
\end{aligned}
\tag{3.19}
$$

The structure function $g_4^{\mathrm{W^-N}}$ is obtained from $g_4^{\mathrm{W^+N}}$ by $A_a^i \to -A_a^i$ $(i=1,2)$.
From the definitions of the matrix elements in Eq. (3.11) one can only obtain results for the following integrals

$$
\begin{aligned}
&\int_{-1}^{1} d\,x V_c^1(x) = V_c^1(0,0)=\Gamma_c\,, \quad \int_{-1}^{1} d\,x A_c^1(x) = A_c^1(0,0)=\Gamma_c^5\,, \\
&\int_{-1}^{1} d\,x \bar{V}_c^1(x) = \bar{V}_c^1(0,0)=0\,, \quad \int_{-1}^{1} d\,x \bar{A}_c^1(x) = \bar{A}_c^1(0,0)=0\,.
\end{aligned}
\tag{3.20}
$$

In order to compute the sum rules which are of the type $\int_0^1 dx\ \Delta F^N(x,q^2)$ one has to convert integrals of the form $\int_{-1}^1 dx\ J_c^k(x)$ into $\int_0^1 dx\ J_c^k(x)$. This is only possible for the following integrals

$$
\begin{aligned}
&\int_{-1}^{1} dx\, V_c^1(x) = \frac{1}{2}\int_0^1 dx\, V_c^1(x)\,, \\
&\int_{-1}^{1} d\,x \left(A_c^1(x)+\frac{\partial A_c^2(x)}{\partial x}\right) = \frac{1}{2}\int_0^1 d\,x\, A_c^1(x)\,,
\end{aligned}
\tag{3.21}
$$

where we have used the property

$$
\int_{-1}^{1} d\,x \frac{\partial J_c^k(x)}{\partial x} = 0\,, \qquad J = V, \bar{V}, A, \bar{A}\,.
\tag{3.22}
$$

However because of the symmetry properties in Eq. (3.13) we obtain

$$
\begin{aligned}
&\int_{-1}^{1} dx\, \bar{V}_c^1(x) = \int_0^1 dx\, \bar{V}_c^1(x) + \int_{-1}^0 dx\, \bar{V}_c^1(x) = \int_0^1 dx\, \bar{V}_c^1(x) + \int_0^1 dx\, \bar{V}_c^1(-x) \\
&= \int_0^1 dx\, \bar{V}_c^1(x) - \int_0^1 dx\, \bar{V}_c^1(x) = 0\,,
\end{aligned}
\tag{3.23}
$$

so that one cannot express $\int_0^1 dx\,\bar V_c^1(x)$ into $\int_{-1}^1 dx\,\bar V_c^1(x)$. The same holds for $\int_0^1 d\,x\,\bar A_c^1(x)$. Therefore we can only compute those sum rules when the combination of structure functions can be either expressed into $V_c^1(x)$ (unpolarized scattering) or into $A_c^1(x)+\frac{\partial A_c^2(x)}{\partial x}$ (polarized scattering).
Hence for charged current interactions only the following fundamental sum rules can be derived. They are given by

unpolarized Bjorken sum rule [3]

$$\int_0^1 d\,x\,\Big(F_1^{\mathrm{W^-N}}(x,q^2)-F_1^{\mathrm{W^+N}}(x,q^2)\Big)$$
$$=(I_3^N+\frac{3}{2}\,Y^N)+\cos^2\theta_c\,(I_3^N-\frac{3}{2}\,Y^N)\,, \tag{3.24}$$

Gross Llewellyn Smith sum rule [4]

$$\int_0^1 d\,x\,\Big(F_3^{\mathrm{W^-N}}(x,q^2)+F_3^{\mathrm{W^+N}}(x,q^2)\Big)$$
$$=(2\,I_3^N-Y^N+4\,B^N)+\cos^2\theta_c\,(-2\,I_3^N+3\,Y^N)\,. \tag{3.25}$$

Further we have the polarized analogue of the Bjorken sum rule in Eq. (3.24)

$$\int_0^1 d\,x\,\Big(g_5^{\mathrm{W^-N}}(x,q^2)-g_5^{\mathrm{W^+N}}(x,q^2)\Big)$$
$$=\Big\{-I_3^N(F+D)-\frac{1}{2}\,(3\,F-D)\Big\}+\cos^2\theta_c\Big\{-I_3^N(F+D)+\frac{1}{2}(3\,F-D)\Big\}\,. \tag{3.26}$$

Because of the Callan-Gross relation mentioned below Eq. (3.11) the Adler sum rule in Eq. (2.24) and its polarized analogue in Eq. (2.26) follow automatically from Eq. (3.24) and Eq. (3.26) respectively. Another important feature is that the Gross-Llewellyn Smith sum rule is determined by the symmetric structure constants d_{abc} whereas the other charged current sum rules are determined by the structure constants f_{abc} of the Lie-algebra of $SU(3)_F$.
For the flavour group $SU(4)_F$ (see Appendix A) we obtain four additional sum rules which are not present in the case of $SU(3)_F$. They are given by

$$\int_0^1 \frac{d\,x}{x}\Big(F_2^{\mathrm{W^\pm p}}(x,q^2)-F_2^{\mathrm{W^\pm n}}(x,q^2)\Big)=\mp 2\,, \tag{3.27}$$

and [1]

$$\int_0^1 \frac{d\,x}{x}\Big(g_4^{\mathrm{W^\pm p}}(x,q^2)-g_4^{\mathrm{W^\pm n}}(x,q^2)\Big)=\pm 2\,(F'+D')\,. \tag{3.28}$$

[1]Very often Eq. (3.27) is called the Adler sum rule. However this is wrong. The correct one is given in Eq. (2.24)

The other two sum rules follow from the Callan-Gross relation [27] and they are obtained from the two above via the replacements $F_2 \rightarrow 2xF_1$ and $g_4 \rightarrow 2xg_5$. Notice that in the derivation of the expressions above we have used isospin symmetry which implies $V^k_{3,p}(x) = -V^k_{3,n}(x)$ and $V^k_{c,p}(x) = V^k_{c,n}(x)$ for $c \neq 3$. In the case of $SU(3)_F$, the expressions in Eqs. (3.27), (3.28) cannot be derived because the combination of structure functions still contain quantities of the type $\bar{V}^1_3(x)$, $\bar{A}^k_3(x)$ $(k = 1, 2)$. Finally note that F' and D' in Eq. (3.28) differ from F and D in Eqs (2.26), (3.26) since the latter are only measured when we assume a $SU(3)_F$ symmetry.

With the help of the light cone algebra we can also derive sum rules for the structure functions measured in neutral current processes. If we define $s = \sin\theta_W$, $c = \cos\theta_W$, where θ_W denotes the weak angle, the neutral electroweak currents for $SU(3)_F$ are given by

$$J^\mu_\gamma(y) = V^\mu_\gamma(y)\,,$$

$$J^\mu_Z(y) = (1 - 2\,s^2)\,V^\mu_\gamma(y) - A^\mu_\gamma(y) - \frac{1}{3}\Big(V^\mu_0(y) - A^\mu_0(y)\Big)\,,$$

$$\text{with} \quad V^\mu_\gamma(y) \equiv V^\mu_3(y) + \frac{1}{3}\sqrt{3}\,V^\mu_8(y) \quad A^\mu_\gamma(y) \equiv A^\mu_3(y) + \frac{1}{3}\sqrt{3}\,A^\mu_8(y)\,. \tag{3.29}$$

Substitution of these currents into the hadronic structure tensor in Eq. (2.2) yields the following results

$$W^{\mu\nu}_{\gamma\gamma} = W^{\mu\nu}_{33,VV} + \frac{2}{3}\sqrt{3}\,W^{\mu\nu}_{\{38\}} + \frac{1}{3}\,W^{\mu\nu}_{88,VV}\,, \tag{3.30}$$

$$\begin{aligned} W^{\mu\nu}_{ZZ} = {} & 2(1 - 2\,s^2 + 2\,s^4)\Big[W^{\mu\nu}_{33,VV} + \frac{2}{3}\sqrt{3}\,W^{\mu\nu}_{\{38\},VV} + \frac{1}{3}\,W^{\mu\nu}_{88,VV}\Big] \\ & -\frac{4}{3}(1 - s^2)\Big[W^{\mu\nu}_{\{30\},VV} + \frac{1}{3}\sqrt{3}\,W^{\mu\nu}_{\{80\},VV}\Big] + \frac{2}{9}\,W^{\mu\nu}_{00,VV} \\ & -2(1 - 2\,s^2)\Big[W^{\mu\nu}_{33,AV} + \frac{2}{3}\sqrt{3}\,W^{\mu\nu}_{\{38\},AV} + \frac{1}{3}\,W^{\mu\nu}_{88,AV}\Big] \\ & +\frac{4}{3}(1 - s^2)\Big[W^{\mu\nu}_{\{30\},AV} + \frac{1}{3}\sqrt{3}\,W^{\mu\nu}_{\{80\},AV}\Big] - \frac{2}{9}\,W^{\mu\nu}_{00,AV}\,, \end{aligned} \tag{3.31}$$

$$\begin{aligned} W^{\mu\nu}_{\gamma Z + Z\gamma} = {} & 2(1 - 2\,s^2)\Big[W^{\mu\nu}_{33,VV} + \frac{2}{3}\sqrt{3}\,W^{\mu\nu}_{\{38\},VV} + \frac{1}{3}\,W^{\mu\nu}_{88,VV}\Big] - \frac{2}{3}\Big[W^{\mu\nu}_{\{30\},VV} \\ & +\frac{1}{3}\sqrt{3}\,W^{\mu\nu}_{\{80\},VV}\Big] - 2\Big[W^{\mu\nu}_{33,AV} + \frac{2}{3}\sqrt{3}\,W^{\mu\nu}_{\{38\},AV} + \frac{1}{3}\,W^{\mu\nu}_{88,AV}\Big] \\ & +\frac{2}{3}\Big[W^{\mu\nu}_{\{30\},AV} + \frac{1}{3}\sqrt{3}\,W^{\mu\nu}_{\{80\},AV}\Big]\,. \end{aligned} \tag{3.32}$$

Since the above structure tensor only involve anti-commutators, indicated by {}, the structure functions are determined by the values for d_{abc}. Let us introduce the following shorthand notations defined by

$$
\begin{aligned}
V_\gamma(x) &= V_3^1(x) + \frac{1}{3}\sqrt{3}V_8^1(x)\,, \\
A_\gamma(x) &= A_3^1(x) + \frac{\partial A_3^2(x)}{\partial x} + \frac{1}{3}\sqrt{3}\left(A_8^1(x) + \frac{\partial A_8^2(x)}{\partial x}\right)\,, \\
V_{\gamma\gamma}(x) &= \frac{4}{9}\,V_0^1(x) + \frac{1}{3}\,V_3^1(x) + \frac{1}{9}\sqrt{3}\,V_8^1(x)\,, \\
A_{\gamma\gamma}(x) &= \frac{4}{9}\left(A_0^1(x) + \frac{\partial A_0^2(x)}{\partial x}\right) + \frac{1}{3}\left(A_3^1(x) + \frac{\partial A_3^2(x)}{\partial x}\right) \\
&\quad + \frac{1}{9}\sqrt{3}\left(A_8^1(x) + \frac{\partial A_8^2(x)}{\partial x}\right)\,.
\end{aligned}
\tag{3.33}
$$

The neutral current structure functions read as follows

$$F_2^{\gamma\mathrm{N}}(x) = x\,\bar{V}_{\gamma\gamma}(x)\,, \tag{3.34}$$

$$g_1^{\gamma\mathrm{N}}(x) = \frac{1}{2}\,A_{\gamma\gamma}(x)\,, \tag{3.35}$$

$$F_2^{\mathrm{ZN}}(x) = 2\,x\,(1-2\,s^2+2\,s^4)\,\bar{V}_{\gamma\gamma}(x) - \frac{2}{3}\,x\,(1-s^2)\,\bar{V}_\gamma(x) + \frac{1}{9}\,x\,\bar{V}_0^1(x)\,, \tag{3.36}$$

$$F_3^{\mathrm{ZN}}(x) = 2\,(1-2\,s^2)\,V_{\gamma\gamma}(x) - \frac{2}{3}(1-s^2)\,V_\gamma(x) + \frac{1}{9}\,V_0^1(x)\,, \tag{3.37}$$

$$
\begin{aligned}
g_1^{\mathrm{ZN}}(x) &= (1-2\,s^2+2\,s^4)\,A_{\gamma\gamma}(x) - \frac{1}{3}(1-s^2)\,A_\gamma(x) \\
&\quad + \frac{1}{18}\left(A_0^1(x) + \frac{\partial A_0^2(x)}{\partial x}\right)\,,
\end{aligned}
\tag{3.38}
$$

$$
\begin{aligned}
g_4^{\mathrm{ZN}}(x) &= -2\,x\,(1-2\,s^2)\,\bar{A}_{\gamma\gamma}(x) + \frac{2}{3}\,x\,(1-s^2)\,\bar{A}_\gamma(x) \\
&\quad - \frac{1}{9}\,x\left(\bar{A}_0^1(x) + \frac{\partial \bar{A}_0^2(x)}{\partial x}\right)\,,
\end{aligned}
\tag{3.39}
$$

$$F_2^{\gamma\mathrm{Z,N}}(x) = 2\,x\,(1-2\,s^2)\,\bar{V}_{\gamma\gamma}(x) - \frac{1}{3}\,x\,\bar{V}_\gamma(x)\,, \tag{3.40}$$

$$F_3^{\gamma\mathrm{Z,N}}(x) \;=\; 2\,V_{\gamma\gamma}(x) - \frac{1}{3}\,V_\gamma(x)\,, \tag{3.41}$$

$$g_1^{\gamma\mathrm{Z,N}}(x) \;=\; (1-2\,s^2)\,A_{\gamma\gamma}(x) - \frac{1}{6}\,A_\gamma(x)\,, \tag{3.42}$$

$$g_4^{\gamma\mathrm{Z,N}}(x) \;=\; -2\,x\,\bar{A}_{\gamma\gamma}(x) + \frac{1}{3}\,x\,\bar{A}_\gamma(x)\,. \tag{3.43}$$

If the proton is replaced by the neutron, the structure functions of the latter are derived from the former via the substitution $J_3^i \to -J_3^i$ with $i = 1, 2$ and $J = V, \bar{V}, A, \bar{A}$. As has been mentioned below Eq. (3.23), we can derive sum rules when the structure functions contain the matrix elements of $V_c^1(x)$ and $A_c^1(x) + \frac{\partial A_c^2(x)}{\partial x}$ only. The results are given by

$$\int_0^1 dx\, F_3^{\mathrm{ZN}}(x,q^2) \;=\; \frac{3}{2}\,(1-2\,s^2)\,B^N - \frac{1}{3}\,s^2\,(2\,I_3^N + S^N)\,, \tag{3.44}$$

$$\int_0^1 dx\, F_3^{\gamma\mathrm{Z,N}}(x,q^2) \;=\; \frac{3}{2}\,B^N + \frac{1}{6}\,(2\,I_3^N + S^N)\,. \tag{3.45}$$

For the longitudinal spin structure function g_1 we obtain

$$\int_0^1 dx\, g_1^{\gamma\mathrm{N}}(x,q^2) \;=\; f_\gamma^{\mathrm{S}} + f^{\mathrm{NS}}\,, \tag{3.46}$$

$$\int_0^1 dx\, g_1^{\mathrm{ZN}}(x,q^2) \;=\; f_{\mathrm{Z}}^{\mathrm{S}} - 2\,s^2(1-2\,s^2)\,f^{\mathrm{NS}}\,, \tag{3.47}$$

$$\int_0^1 dx\, g_1^{\gamma\mathrm{Z,N}}(x,q^2) \;=\; f_{\gamma\mathrm{Z}}^{\mathrm{S}} + (1-4\,s^2)\,f^{\mathrm{NS}}\,, \tag{3.48}$$

$$f^{\mathrm{NS}} \;=\; \frac{1}{6}\,I_3^N\,(F+D) + \frac{1}{36}\,(3\,F-D)\,, \tag{3.49}$$

$$f_\gamma^{\mathrm{S}} \;=\; \frac{2}{9}\,\Gamma_0^{5,N}\,,$$

$$f_{\mathrm{Z}}^{\mathrm{S}} \;=\; (\frac{4}{9} - \frac{8}{9}\,s^2 + \frac{8}{9}s^4)\,\Gamma_0^{5,N}\,,$$

$$f_{\gamma Z}^{\mathrm{S}} \;=\; \frac{4}{9}(1-2\,s^2)\,\Gamma_0^{5,N}\,. \tag{3.50}$$

Here the superscripts S and NS refer to the singlet and non-singlet representations of the flavour group $SU(3)_F$. Contrary to structure function F_3 in Eqs. (3.44), (3.45), the sum rules for g_1 Eqs. (3.46)-(3.48) cannot be expressed into the quantum numbers of the hadron because $\Gamma_0^{5,N}$ in Eq. (3.50) is unknown. Expressions (3.46)-(3.48) are

generalizations of the Ellis-Jaffe sum rule for electro-weak currents which was originally derived for electro-production [20]. Moreover the singlet axial-vector current $A_0^\mu(y)$ leading to the matrix element $A_0^1(x) + \frac{\partial A_0^2(x)}{\partial x}$ in Eq. (3.33) is not conserved due to the Adler-Bell-Jackiw (ABJ)anomaly [21]. Therefore $\Gamma_0^{5,N}$ will become scale dependent when higher order QCD corrections are included. Hence the Ellis-Jaffe sum rule is non-fundamental and it is no surprise that it is violated experimentally [29]. However using isospin symmetry one can derive the generalization of the polarized Bjorken sum rule [2] given by the expressions

$$\int_0^1 dx \left(g_1^{\gamma p}(x,q^2) - g_1^{\gamma n}(x,q^2) \right) = \frac{1}{6}\,(F+D)\,, \tag{3.51}$$

$$\int_0^1 dx \left(g_1^{Zp}(x,q^2) - g_1^{Zn}(x,q^2) \right) = -\frac{1}{3}\,s^2(1-2\,s^2)\,(F+D)\,, \tag{3.52}$$

$$\int_0^1 dx \left(g_1^{\gamma Z,p}(x,q^2) - g_1^{\gamma Z,n}(x,q^2) \right) = \frac{1}{6}\,(1-4\,s^2)\,(F+D)\,. \tag{3.53}$$

When in the Bjorken limit the leading contributions are coming from twist two operators only, the structure functions can be expressed into parton densities which become scale dependent when higher order QCD corrections are included. Hence one obtains relations between all bilocal operator matrix elements and the parton densities so that one can also compute sum rules involving the matrix elements $\bar V_c^k(x)$ or $\bar A_c^k(x)$. However the integrals $\int_0^1 dx\ \bar V_c^k(x)$, $\int_0^1 dx\ \bar A_c^k(x)$ can only be computed if one makes additional model dependent assumptions. Moreover it turns out they receive scaling violating contributions as e.g. is observed for the Ellis-Jaffe sum rule (see below Eq. (3.50)). Therefore it is no surprise that the results are very often in disagreement with experiment as we will show in an example below. Therefore these sum rules will be called non-fundamental or parton model sum rules. The relations between the bilocal operator matrix elements and the unpolarized parton densities $q(x)$ are given by

$$\begin{aligned}
V_3^1(x) &= \frac{1}{2}\Big(V_u(x) - V_d(x)\Big)\,,\\
V_8^1(x) &= \frac{1}{2\sqrt{3}}\Big(V_u(x) + V_d(x) - 2\,V_s(x)\Big)\,,\\
V_0^1(x) &= \frac{1}{2}\Big(V_u(x) + V_d(x) + V_s(x)\Big)\,,\\
\bar V_3^1(x) &= \frac{1}{2}\Big(\Delta_u(x) - \Delta_d(x)\Big)\,,\\
\bar V_8^1(x) &= \frac{1}{2\sqrt{3}}\Big(\Delta_u(x) + \Delta_d(x) - 2\,\Delta_s(x)\Big)\,,
\end{aligned}$$

$$\bar{V}_0^1(x) \;=\; \frac{1}{2}\Sigma(x)\,. \tag{3.54}$$

Here the non-singlet $V_q(x)$, $\Delta_q(x)$ and the singlet $\Sigma(x)$ parton densities are defined by

$$V_q(x) = q(x) - \bar{q}(x)\,, \qquad \Delta_q(x) = q(x) + \bar{q}(x) - \frac{1}{3}\Sigma(x)\,,$$

$$\Sigma(x) = \sum_{q=u,d,s} q(x) + \bar{q}(x)\,. \tag{3.55}$$

For the longitudinally polarized parton densities $\delta q(x)$ we obtain

$$\begin{aligned}
A_3^1(x) + \frac{\partial A_3^2(x)}{\partial x} &= \frac{1}{2}\Big(\Delta_{\delta u}(x) - \Delta_{\delta d}(x)\Big)\,,\\
A_8^1(x) + \frac{\partial A_8^2(x)}{\partial x} &= \frac{1}{2\sqrt{3}}\Big(\Delta_{\delta u}(x) + \Delta_{\delta d}(x) - 2\,\Delta_{\delta s}(x)\Big)\,,\\
A_0^1(x) + \frac{\partial A_0^2(x)}{\partial x} &= \frac{1}{2}\delta\Sigma(x)\,,\\
\bar{A}_3^1(x) + \frac{\partial \bar{A}_3^2(x)}{\partial x} &= \frac{1}{2}\Big(V_{\delta u}(x) - V_{\delta d}(x)\Big)\,,\\
\bar{A}_8^1(x) + \frac{\partial \bar{A}_8^2(x)}{\partial x} &= \frac{1}{2\sqrt{3}}\Big(V_{\delta u}(x) + V_{\delta d}(x) - 2\,V_{\delta s}(x)\Big)\,,\\
\bar{A}_0^1(x) + \frac{\partial \bar{A}_0^2(x)}{\partial x} &= \frac{1}{2}\Big(V_{\delta u}(x) + V_{\delta d}(x) + V_{\delta s}(x)\Big)\,,
\end{aligned} \tag{3.56}$$

with similar notations as in Eq. (3.55) for the polarized quark densities. If one includes higher order QCD corrections all parton densities will depend on a scale except if one takes the first moment represented by the integrals of the type $\int_0^1 dx\; V_c^1(x)$ or $\int_0^1 dx\; A_c^1(x)$. This is because the integrals are related to conserved vector and axial-vector currents respectively. Hence from Eqs. (3.54), (3.56) it follows that $\int_0^1 dx\, V_q(x)$ and $\int_0^1 dx\, \Delta_{\delta q}(x)$ are scale independent This does not hold for $\int_0^1 dx\, \Delta_q(x)$ and $\int_0^1 dx\, V_{\delta q}(x)$. Here the scale dependence is ruled by an anomalous dimension which becomes non-vanishing in order α_s^2 (see [30]). This for instance happens in the Gottfried sum rule [22] given by

$$\int_0^1 \frac{d\,x}{x}\left(F_2^{\gamma \mathrm{p}}(x,q^2) - F_2^{\gamma n}(x,q^2)\right) = \frac{2}{3}\int_0^1 dx\,\bar{V}_3^1(x)\,, \tag{3.57}$$

where $\bar{V}_3^1(x) = (\Delta_u(x) - \Delta_d(x))/2$ (see Eq. (3.54)). Hence this expression acquires second order QCD corrections containing scaling violating terms. Furthermore the right-hand side is model dependent because it is not related to the expectation value of

a conserved (axial-) vector current. Only under certain assumptions i.e. $\bar{u}(x) = \bar{d}(x)$ the above integral yields 1/3. Because of the scaling violating term and the model dependence it is no surprise that this result is in disagreement with experiment (see e.g [31]). Hence one can conclude that all sum rules which cannot be related to the expectation values of (axial-) vector currents are model dependent and show scaling violating terms in the perturbation series due to non-vanishing anomalous dimensions. These sum rules are non-fundamental and they are not suitable as a test of perturbative QCD. A collection of these parton model sum rules is shown in table 2 of [15].

There is a second class of non-fundamental sum rules which originate from structure functions which contain besides twist two also twist three contributions. They only appear in polarized scattering and hold for charged current as well as neutral current processes. These sum rules can be derived from Eq. (3.10) and Eq. (3.22). The most well known one is the Burkhardt-Cottingham sum rule given by [23]

$$\int_0^1 dx\, g_{2,ab}^{VV}(x,q^2) = \int_0^1 dx\, g_{2,ab}^{AA}(x,q^2) = 0\,. \tag{3.58}$$

However the sum rule above is not the only one which equals zero. In a similar way one can also derive that

$$\int_0^1 dx\, \left(g_{4,ab}^{AV}(x,q^2) - g_{3,ab}^{AV}(x,q^2)\right) = 0\,, \tag{3.59}$$

$$\int_0^1 dx\, \left(2\,x\, g_{5,ab}^{AV}(x,q^2) - g_{3,ab}^{AV}(x,q^2)\right) = 0\,, \tag{3.60}$$

yield zero. For charged current interactions where only the expectation value of the axial vector current shows up one obtains

$$\int_0^1 \frac{dx}{x}\, \left(g_{3,[ab]}^{AV}(x,q^2) + 2\,x\, g_{9,[ab]}^{AV}(x,q^2)\right) = \, f_{abc}\,\Gamma_c^5\,, \tag{3.61}$$

$$\int_0^1 \frac{dx}{x}\, \left(g_{3,[ab]}^{AV}(x,q^2) - 2\,x\, g_{8,[ab]}^{AV}(x,q^2)\right) = \, f_{abc}\,\Gamma_c^5\,. \tag{3.62}$$

They look like the expressions derived in Eq. (2.14) and in the next section we will study whether they receive QCD and power corrections. Notice that the last two sum rules are hard to measure since they contain the structure functions g_8, g_9 which do not show up in the cross section when lepton masses are neglected.

4 QCD and power corrections to the sum rules

In this section we will discuss how the sum rules derived in the previous section are modified by QCD and power corrections. We will show that when power corrections occur the sum rule also receives QCD corrections. This statement only holds for twist two sum rules. At the end we also discuss sum rules which receive twist three contributions. All sum rules derived in the previous sections can be written as

$$\int_0^1 dx\, \Delta F_i(x,q^2,m^2) = \sum_a \Gamma_a^N\, \mathcal{C}_{i,q}\left(\alpha_s(\mu^2), \frac{q^2}{m^2}\right) \quad + \quad \text{higher twist}\,. \tag{4.1}$$

The above form follows from the operator expansion where Γ_a^N is the expectation value of a vector current sandwiched between the hadronic state $N(p,s)$ (see Eq. (2.10)). In the case of an axial-vector current Γ_a^N is replaced by $\Gamma_a^{5,N}$. The quantity $\mathcal{C}_{i,q}$ denotes the first moment of the quark non-singlet coefficient function and it contains all QCD and power corrections of the type m^2/q^2. Notice that vector currents are conserved so that they are not renormalized. However axial-vector currents are partially conserved which means that they receive finite renormalizations. Examples of this phenomenon are the isospin currents V_3^μ and A_3^μ. The expectation values read

$$\Gamma_3^N = 2\, I_3^N\, g_V\,, \qquad \Gamma_3^{5,N} = 2\, I_3^N\, g_A\,. \tag{4.2}$$

Here we have $g_V = 1$ but $g_A = F + D \neq 1$ (see below Eq. (2.27)). In both cases Γ_a^N as well as $\Gamma_a^{5,N}$ are scale independent so that all power corrections can be either attributed to $\mathcal{C}_q$ or to higher twist. When the singlet axial-vector current shows up, as happens for the Ellis-Jaffe sum rule, the corresponding expectation value $\Gamma_0^{5,N}$ becomes scale dependent and the coefficient function receives logarithmic corrections [36]. This is because the symmetry is so badly broken by the ABJ-anomaly [21] that the quantity A_0^μ will receive infinite renormalizations.

For unpolarized scattering the first moment of the coefficient function, presented on the right-hand side of Eq. (4.1), is given by

$$\int_0^1 dz\, \hat{\mathcal{F}}_i(z,q^2,m^2) = \Gamma_q\, \mathcal{C}_{i,q}(\alpha_s, \frac{q^2}{m^2})\,. \tag{4.3}$$

In the case of polarized scattering we have

$$\int_0^1 dz\, \hat{g}_i(z,q^2,m^2) = \Gamma_q^5\, \delta\mathcal{C}_{i,q}(\alpha_s, \frac{q^2}{m^2})\,. \tag{4.4}$$

Here $\hat{\mathcal{F}}_i$ and $\hat{g}_i$ are the partonic structure functions which depend on the mass m, virtuality q^2 of the vector-boson and the partonic scaling variable $z = -q^2/2p\cdot q$ where p now stands for the momentum of the incoming quark. The partonic quantities are defined in the same way as the hadronic structure functions in Eq. (2.3) except that the hadronic currents are replaced by the quark currents. Further we have the properties

$$\Gamma_q = 1\,, \qquad \Gamma_q^5 = \sum_{n=0}^\infty \Gamma_q^{5,n} \left(\frac{\alpha_s(\mu^2)}{4\,\pi}\right)^n\,, \qquad \Gamma_q^{5,0} = 1\,. \tag{4.5}$$

The results above follow from quark current conservation and partial conservation of the quark axial-vector current ($\partial_\mu A_a^\mu \neq 0$) which happens if the quark has a mass $m \neq 0$. In the case the quark is massless the axial vector is conserved too and we have $\Gamma_q^5 = 1$ in all orders of perturbation theory.
The partonic structure functions are computed from the process

$$V + q \rightarrow' partons' \,, \quad \text{with} \quad V = \gamma, Z, W^\pm \,, \tag{4.6}$$

where $'partons'$ represents all (anti-) quarks and gluons which can be produced in the final state. Further the general vertex for the coupling of the vector boson V to the quarks will be denoted by

$$\Gamma_{a,\mu}^{V,(0)} = -i\,(v_a^V + a_a^V \gamma_5)\gamma_\mu \,, \qquad V = \gamma, Z, W \,, \tag{4.7}$$

where

$$\begin{aligned} v_a^\gamma &= Q_a \,, \qquad a_a^\gamma = 0 \,, \\ v_a^Z &= T_a^3 - 2\, s_w^2 Q_a \,, \qquad a_a^Z = T_a^3 \,, \\ &\quad v_a^W = a_a^W = 1 \,. \end{aligned} \tag{4.8}$$

In the case of the quarks the electroweak charges are equal to

$$\begin{aligned} Q_a &= \frac{2}{3}, \quad T_3^a = \frac{1}{2}\,, \quad a = u, c, t \,, \\ Q_a &= -\frac{1}{3}, \quad T_3^a = -\frac{1}{2}\,, \quad a = d, s, b \,. \end{aligned} \tag{4.9}$$

In this section we aim to compute the coefficient function up to order α_s for non-zero masses of the quarks. We only show explicit results for those structure functions which show up in the cross section for vanishing lepton masses which means that no formulae are given for $\hat{\mathcal{F}}_4, \hat{\mathcal{F}}_5$, $\hat{g}_6, \hat{g}_8, \hat{g}_9$. In lowest order we have the process

$$V(q) \;+\; q(p,s) \rightarrow q(p') \,, \quad \text{with} \quad p^2 = p'^2 = m^2 \,, \tag{4.10}$$

which provides us with the Born approximations to the unpolarized and polarized coefficient functions denoted by $\mathcal{C}_{i,q}^{(0)}$ and $\delta\mathcal{C}_{i,q}^{(0)}$ respectively. In this paper the calculations are only performed for neutral current interactions for which $p^2 = p'^2$ [2]. However the results are such that the conclusions also hold for other mass assignments where $p^2 \neq p'^2$ which occurs for charged current processes. There is only one exception. In the case of the mass assignment in Eq. (4.10) it turns out that $g_6^{VV} = 0$ whereas $g_6^{AA} \neq 0$ which is in disagreement with the result found for the bilocal current algebra in Eq. (3.10). This is not surprising because the vector current is conserved in contrast to the

[2]Sum rules for $p^2 = 0$ and $p'^2 = m^2$ are treated up to first order in [32]

axial vector current. We checked that for the choice $p^2 = m^2$ and $p'^2 = 0$, where both currents are not conserved, one obtains the relation $g_6^{VV} = g_6^{AA}$ which is in agreement with Eq. (3.10). For all other structure functions the mass assignment is irrelevant provided one takes the Bjorken limit. In next order we receive contributions from gluon bremsstrahlung given by

$$V(q) \ + \ q(p,s) \rightarrow q(p') + g(k)\,, \tag{4.11}$$

and the virtual corrections to Eq. (4.10). Because of the infrared divergence appearing at $z = 1$ we have to split the integrals in Eqs. (4.3), (4.4) into

$$\begin{aligned}
\int_0^1 dz\, \hat{\mathcal{F}}_{i,q}^{(1)}(z,q^2,m^2) &= \int_0^{z_{max}} dz\, \hat{\mathcal{F}}_{i,q}^{\rm HARD}(z,q^2,m^2) + \int_{z_{max}}^1 dz\, \hat{\mathcal{F}}_{i,q}(z,q^2,m^2,\lambda^2) \\
&\quad + \int_0^1 dz\, \hat{\mathcal{F}}_{i,q}^{\rm VIRT}(z,q^2,m^2,\lambda^2)\,, \\
\hat{\mathcal{F}}_{i,q}^{\rm HARD}(z,q^2,m^2) &\equiv \hat{\mathcal{F}}_{i,q}(z,q^2,m^2,0)\,, \\
z_{max} &= \frac{-q^2}{-q^2 + 2\ m\ \omega}\,, \quad \text{with} \quad \omega \ll m\,,
\end{aligned} \tag{4.12}$$

with a similar expression for Eq. (4.4). Here we have introduced a gluon mass λ in order to regularize the infrared divergence. The computation of the hard gluon, soft gluon and virtual gluon part of the structure functions proceed in the same way as outlined in [33] where the calculation was carried out for $-q^2 \gg m^2$. The computation of the hard gluon part $\mathcal{F}_i^{\rm HARD}$ is straightforward and the results are given in Appendix B. The soft gluon integral is given by

$$\int_{z_{max}}^1 dz\, \hat{\mathcal{F}}_{i,q}(z,q^2,m^2,\lambda^2) = S^{\rm SOFT}(q^2,m^2,\lambda^2,\omega)\, \mathcal{C}_{i,q}^{(0)}\,, \tag{4.13}$$

where $S^{\rm SOFT}$ is given by

$$S^{SOFT} = -\frac{\alpha_s}{8\pi^2} C_F \int_0^\omega \frac{d^3k}{k^0} \left(\frac{m^2}{(p\cdot k)^2} + \frac{m^2}{(p'\cdot k)^2} - \frac{2p\cdot p'}{(p\cdot k)(p'\cdot k)} \right)\,, \tag{4.14}$$

and C_F denotes the colour factor which in QCD equals to 4/3. For $\lambda^2 \ll 2\,m\,\omega \ll m^2$ one obtains the result

$$\begin{aligned}
S^{SOFT} &= \frac{\alpha_s}{4\pi} C_F \Bigg[-2\ln\left(\frac{4\omega^2}{\lambda^2}\right) + 2 - \frac{4m^2 - 2q^2}{\sqrt{q^4 - 4m^2q^2}} \Bigg\{ -\ln\left(\frac{4\omega^2}{\lambda^2}\right)\ln(t) \\
&\quad -2\mathcal{L}i_2\left(\frac{1}{t}\right) - 2\mathcal{L}i_2\left(-\frac{1}{t}\right) - 3\ln^2(t) - \ln(t) + 2\ln(t)\ln(t-1) \\
&\quad +2\ln(t)\ln(t+1) + \zeta(2) \Bigg\} \Bigg]\,,
\end{aligned}$$

$$t = \frac{\sqrt{q^4 - 4m^2q^2} - q^2}{\sqrt{q^4 - 4m^2q^2} + q^2}\,. \tag{4.15}$$

Here $\mathcal{L}i_2(z)$ denotes the dilogarithm which is defined in [34]. The virtual gluon part is obtained from the order α_s corrected vector-boson quark vertex given by

$$\begin{aligned}\Gamma^{V,(1)}_{q,\mu} &= -i\Bigg[\gamma_\mu(1+\mathcal{R}_1)\ v_q^V + \gamma_5\gamma_\mu(1+\mathcal{R}_1+2\mathcal{R}_2)\ a_q^V \\ &\quad + \frac{(p_\mu + {p'}_\mu)}{2m}\mathcal{R}_2\ v_q^V + \gamma_5\frac{q_\mu}{2m}\mathcal{R}_3\ a_q^V\Bigg]\,. \end{aligned} \tag{4.16}$$

For $\lambda^2 \ll m^2$ the functions $\mathcal{R}_i$ become equal to

$$\begin{aligned}\mathcal{R}_1 &= \frac{\alpha_s}{4\pi}C_F\Bigg[\left(\frac{4m^2-2q^2}{\sqrt{q^4-4m^2q^2}}\ln(t) - 2\right)\ln\left(\frac{\lambda^2}{m^2}\right) - 4 - 3\frac{\sqrt{q^4-4m^2q^2}}{q^2}\ln(t) \\ &\quad + \frac{4m^2-2q^2}{\sqrt{q^4-4m^2q^2}}\left\{\frac{3}{2}\ln^2(t) - 2\ln(t)\ln(t+1) + 2\mathcal{L}i_2\left(-\frac{1}{t}\right) + \zeta(2)\right\}\Bigg]\,, \\ \mathcal{R}_2 &= \frac{\alpha_s}{4\pi}C_F\Bigg[-\frac{4m^2}{\sqrt{q^4-4m^2q^2}}\ln(t)\Bigg]\,, \\ \mathcal{R}_3 &= \frac{\alpha_s}{4\pi}C_F\Bigg[-\left(3-\frac{4m^2}{q^2}\right)\frac{4m^2}{\sqrt{q^4-4m^2q^2}}\ln(t) - \frac{8m^2}{q^2}\Bigg]\,. \end{aligned} \tag{4.17}$$

The above expressions also provide us with the renormalization constants of the vector and axial vector current since

$$i\,\Gamma^V_{q,\mu}(q) = v_q^V\langle q(p',s)\mid V_\mu \mid q(p,s)\rangle - a_q^V\langle q(p',s)\mid A_\mu\mid q(p,s)\rangle\,. \tag{4.18}$$

For $p = p'$ or $q^2 = 0$ we obtain

$$\Gamma_q = 1 + \mathcal{R}_1 + \mathcal{R}_2 = 1\,, \quad \Gamma_q^5 = 1 + \mathcal{R}_1 + 2\,\mathcal{R}_2 = 1 - \frac{\alpha_s(\mu^2)}{4\,\pi}\,C_F\,\Big\{2\Big\}\,. \tag{4.19}$$

The computation of $\hat{\mathcal{F}}^{\rm VIRT}_{i,q}$ proceeds in the same way as for the Born approximations $\mathcal{F}^{(0)}_{i,q}$. Since they are all proportional to $\delta(1-z)$ the integral in Eq. (4.12) becomes trivial. After adding this result to the soft gluon integral in Eq. (4.13) the infrared regulator λ will be cancelled. The resulting expression is given by

$$\mathcal{T}^{\rm S+V}_{i,q} = S^{\rm SOFT}(q^2,m^2,\lambda^2,\omega)\,\mathcal{C}^{(0)}_{i,q} + \int_0^1 dz\,\hat{\mathcal{F}}^{\rm VIRT}_{i,q}(z,q^2,m^2,\lambda^2)\,. \tag{4.20}$$

Using the expressions for $\hat{\mathcal{F}}^{\rm HARD}_{i,q}$ in Appendix B one can now compute the first integral on the right-hand side in Eq. (4.12). After taking the limit $\omega \to 0$ we obtain the

following sum rules

$$
\begin{aligned}
\mathcal{C}_{1,q} \quad &= \quad \int_0^1 dz \hat{\mathcal{F}}_{1,q}(z,q^2,m^2) = \\
& v_q^{V_1} v_q^{V_2} \Bigg[\frac{1}{2} + \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg\{ \Bigg(-12 + \frac{4q^6}{(m^2+q^2)^3} - \frac{6q^4}{(m^2+q^2)^2} \\
& + \frac{5q^2}{(m^2+q^2)} \Bigg) \ln \frac{t-1}{\sqrt{t}} + \frac{1}{\sqrt{q^4 - 4m^2q^2}} \Bigg(16m^2 - 5q^2 \Bigg) \ln(t) \\
& - \frac{2q^4}{(m^2+q^2)^2} + \frac{2q^2}{(m^2+q^2)} + \Bigg(6m^2 - \frac{q^2}{2} \Bigg) \mathcal{J} \Bigg\} \Bigg] \\
& + a_q^{V_1} a_q^{V_2} \Bigg[\frac{1}{2} \Bigg(1 - \frac{4m^2}{q^2} \Bigg) + \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg\{ \Bigg(-32 + \frac{32m^2}{q^2} \\
& + \frac{4q^6}{(m^2+q^2)^3} - \frac{10q^4}{(m^2+q^2)^2} + \frac{25q^2}{(m^2+q^2)} \Bigg) \ln \frac{t-1}{\sqrt{t}} \\
& + \frac{1}{\sqrt{q^4 - 4m^2q^2}} \Bigg(28m^2 - 7q^2 \Bigg) \ln(t) - 2 - \frac{2q^4}{(m^2+q^2)^2} \\
& - \frac{8m^2}{q^2} + \frac{4q^2}{(m^2+q^2)} - \Bigg(\frac{16m^4}{q^2} - 16m^2 + \frac{q^2}{2} \Bigg) \mathcal{J} \Bigg\} \Bigg] \\
\underset{-q^2 \gg m^2}{=} \quad & v_q^{V_1} v_q^{V_2} \Bigg[\frac{1}{2} + \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg\{ -1 - \Bigg(\frac{m^2}{-q^2} \Bigg) \Bigg(\frac{37}{9} + \frac{10}{3} \ln \Bigg(\frac{m^2}{-q^2} \Bigg) \Bigg) \Bigg\} \Bigg] \\
& + a_q^{V_1} a_q^{V_2} \Bigg[\frac{1}{2} + \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg\{ -1 - \Bigg(\frac{m^2}{-q^2} \Bigg) \Bigg(\frac{91}{9} - \frac{26}{3} \ln \Bigg(\frac{m^2}{-q^2} \Bigg) \Bigg) \Bigg\} \Bigg], \qquad (4.21)
\end{aligned}
$$

$$
\mathcal{C}_{2,q}^{(1)} \quad = \quad \int_0^1 \frac{dz}{z} \hat{\mathcal{F}}_2(z,q^2,m^2) = v_q^{V_1} v_q^{V_2} + a_q^{V_1} a_q^{V_2} \qquad (4.22)
$$

$$
\begin{aligned}
\mathcal{C}_{3,q} \quad &= \quad \int_0^1 dz \hat{\mathcal{F}}_3(z,q^2,m^2) = \Big(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2} \Big) \Bigg[1 + \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg\{ \Bigg(-16 + \frac{2q^2}{m^2} \\
& + \frac{2q^4}{(m^2+q^2)^2} - \frac{2q^2}{(m^2+q^2)} \Bigg) \ln \frac{t-1}{\sqrt{t}} + \frac{1}{\sqrt{q^4 - 4m^2q^2}} \Bigg(24m^2 + \frac{q^4}{m^2} \\
& - 10q^2 \Bigg) \ln(t) - 4 - \frac{q^2}{(m^2+q^2)} + 8m^2 \mathcal{J} \Bigg\} \Bigg]
\end{aligned}
$$

$$
\underset{-q^2\gg m^2}{=} \left(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\right)\left[1+\frac{\alpha_s(\mu^2)}{4\pi}C_F\left\{-3+\left(\frac{m^2}{-q^2}\right)\left(-4\right.\right.\right.
$$

$$
\left.\left.\left.-3\ln\left(\frac{m^2}{-q^2}\right)\right)\right\}\right], \tag{4.23}
$$

$$
\begin{aligned}
\delta\mathcal{C}_{1,q} &= \left(\Gamma_q^5\right)^{-1}\int_0^1 dz\hat{g}_1(z,q^2,m^2) = v_q^{V_1}v_q^{V_2}\left[\frac{1}{2}+\frac{\alpha_s(\mu^2)}{4\pi}C_F\left\{\left(-8-\frac{q^2}{m^2}\right.\right.\right. \\
&\left.+\frac{q^2}{2(m^2+q^2)}\right)\ln\frac{t-1}{\sqrt{t}}+\frac{1}{\sqrt{q^4-4m^2q^2}}\left(14m^2-\frac{q^4}{2m^2}-\frac{5q^2}{2}\right)\ln(t) \\
&\left.\left.-1+\left(4m^2+\frac{q^2}{4}\right)\mathcal{J}\right\}\right] \\
&+a_q^{V_1}a_q^{V_2}\left[\frac{1}{2}+\frac{\alpha_s(\mu^2)}{4\pi}C_F\left\{\left(-\frac{q^2}{m^2}+\frac{2q^4}{(m^2+q^2)^2}+\frac{q^2}{2(m^2+q^2)}\right)\ln\frac{t-1}{\sqrt{t}}\right.\right. \\
&+\frac{1}{\sqrt{q^4-4m^2q^2}}\left(-2m^2-\frac{q^4}{2m^2}+\frac{5q^2}{2}\right)\ln(t) \\
&\left.\left.-\frac{q^2}{(m^2+q^2)}+\frac{q^2}{4}\mathcal{J}\right\}\right] \\
&\underset{-q^2\gg m^2}{=} v_q^{V_1}v_q^{V_2}\left[\frac{1}{2}+\frac{\alpha_s(\mu^2)}{4\pi}C_F\left\{-\frac{3}{2}+\left(\frac{m^2}{-q^2}\right)\left(\frac{5}{9}-\frac{11}{6}\ln\left(\frac{m^2}{-q^2}\right)\right)\right\}\right] \\
&+a_q^{V_1}a_q^{V_2}\left[\frac{1}{2}+\frac{\alpha_s(\mu^2)}{4\pi}C_F\left\{-\frac{3}{2}-\left(\frac{m^2}{-q^2}\right)\left(\frac{22}{9}+\frac{11}{6}\ln\left(\frac{m^2}{-q^2}\right)\right)\right\}\right], \qquad (4.24)
\end{aligned}
$$

$$
\delta\mathcal{C}_{4,q} = \left(\Gamma_q^5\right)^{-1}\int_0^1\frac{dz}{z}\hat{g}_4(z,q^2,m^2) = -\left(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\right), \tag{4.25}
$$

$$
\begin{aligned}
\delta\mathcal{C}_{5,q} &= \left(\Gamma_q^5\right)^{-1}\int_0^1 dz\hat{g}_5(z,q^2,m^2) = \left(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\right)\left[-\frac{1}{2}+\frac{\alpha_s(\mu^2)}{4\pi}C_F\left\{\right.\right. \\
&\left(8-\frac{q^4}{(m^2+q^2)^2}+\frac{3q^2}{2(m^2+q^2)}\right)\ln\frac{t-1}{\sqrt{t}}+\frac{1}{\sqrt{q^4-4m^2q^2}}\left(-14m^2\right. \\
&\left.\left.\left.+\frac{7}{2}q^2\right)\ln(t)+2+\frac{q^2}{2(m^2+q^2)}-\left(4m^2+\frac{3q^2}{4}\right)\mathcal{J}\right\}\right]
\end{aligned}
$$

$$\underset{-q^2\gg m^2}{=} \left(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\right)\Bigg[-\frac{1}{2}+\frac{\alpha_s(\mu^2)}{4\pi}C_F\Bigg\{1+\left(\frac{m^2}{-q^2}\right)\Bigg(\frac{4}{3}$$

$$+2\ln\left(\frac{m^2}{-q^2}\right)\Bigg)\Bigg\}\Bigg], \tag{4.26}$$

where the integral $\mathcal{J}$ is defined as

$$\begin{aligned}
\mathcal{J} &= \int_{m^2}^{\infty} d\hat{s}\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\ln\left(\frac{\hat{s}+m^2-q^2-\sqrt{\lambda}}{\hat{s}+m^2-q^2+\sqrt{\lambda}}\right)\\
&= \frac{1}{2m\sqrt{-q^2}}\Bigg[-4\mathcal{L}i_2\left(1-\sqrt{t}\right)-4\mathcal{L}i_2\left(-\sqrt{t}\right)+2\mathcal{L}i_2\left(-\frac{t-1}{\sqrt{t}}\right)\\
&\quad -2\mathcal{L}i_2\left(1-\frac{\sqrt{t}}{t-1}\right)-2\ln(t)\ln(1+\sqrt{t})+\frac{1}{4}\ln^2(t)-\ln^2(t-1)\\
&\quad -\ln(t)\ln(t+\sqrt{t}-1)+2\ln(t-1)\ln(t+\sqrt{t}-1)-4\zeta(2)\Bigg].
\end{aligned} \tag{4.27}$$

The result for $\mathcal{C}_{3,q}$ in Eq. (4.23) and the vector-vector part of $\delta\mathcal{C}_{1,q}$ in Eq. (4.24) is already presented in [35]. The reason that the sum rules in Eqs. (4.22), (3.25) do not receive order α_s corrections is a consequence of the ETC algebra presented in Eq. (2.5). This is because the order α_s coefficient functions are the same for charged current as well as neutral current interactions. However in next order this property does not hold anymore. From [36], [37] one can infer that the coefficient functions are different due to the sign of the contributions coming from processes with equal quarks in the final state given by the reaction $V+q\rightarrow q+q+\bar{q}$. It turns out that sum rules corresponding to charged current interactions interactions do not receive order α_s^2 corrections which is the case for the Adler sum rule in Eq. (2.24) and its polarized analogue in Eq. (2.26). Notice that this statement is verified for $m=0$ but it has to hold for $m\neq 0$ as well. However for the neutral current sum rules like the Gottfried sum rule in Eq. (3.57) one obtains order α_s^2 corrections which even contain scaling violating terms of the type $\ln -q^2/m^2$ which have to be absorbed in Γ_q (see also below Eq. (3.57)). Therefore one should be careful in drawing to premature conclusions about the vanishing of the order α_s corrections to sum rules. From the computations above we also observe that if a sum rule gets order α_s corrections it also receives power corrections and vice versa. These power corrections are of the type m^2/q^2 and they are a signature of higher twist contributions. Further in the limit $m\rightarrow 0$ we have the following relations which hold up to order α_s^2 (see [5], [6], [36], [37])

$$\mathcal{C}_{1,q}=\delta\mathcal{C}_{5,q}\,,\qquad \mathcal{C}_{3,q}=\delta\mathcal{C}_{1,q}\,. \tag{4.28}$$

These relations are broken in order α_s^3. In [6] one has found that in this order $\mathcal{C}_{3,q}\neq\delta\mathcal{C}_{1,q}$. Probably this also holds for $\mathcal{C}_{1,q}$, $\delta\mathcal{C}_{5,q}$ although the third order result for the

latter is not known yet. We also want to emphasize that the correct expressions for the coefficient functions are only obtained if the renormalization of the axial-vector current given by Γ_q^5 is taken into account. This is sometimes forgotten in the literature (see e.g. [35], [38]). Hence it is incorrect to conclude that in the limit $-q^2 \gg m^2$ the polarized coefficient functions $\delta\mathcal{C}_{i,q}$ depend on the choice of the mass i.e. $m = 0$ versus $m \neq 0$. In the case $m = 0$, where $\Gamma_q^5 = 1$ (see the remark below Eq. (4.5)), one can identify the first moments of the partonic structure function and the coefficient functions but for $m \neq 0$ this is not allowed. Another remark we want to make is that one should be careful with n-dimensional regularization when the γ_5 matrix is present. There exist prescriptions which break the conservation of the non-singlet axial-vector current. To restore the Ward identities one needs an additional renormalization constant which is the analogue of Γ_q^5 introduced in the equations above. In this case the renormalization constant can even become infinite (see e.g. [6]).

Next we discuss the sum rules, derived at the end of section 3, which also involve leading twist three contributions. The first one is the Burkhardt-Cottingham sum rule [23] in Eq. (3.58) Using Eqs. (B.5) and (B.16) one obtains up to order α_s

$$\int_0^1 dz\, \hat{g}_2(z, q^2, m^2) = 0\,. \tag{4.29}$$

This result was already checked for the vector-vector part in the context of QED in [39] (see also [40], [41]). In this paper we have shown that it also holds for the axial-vector-axial-vector part. The second sum rule presented in Eq. (3.59) also yields zero up to next to leading order

$$\int_0^1 dz\, \Big(\hat{g}_4(z, q^2, m^2) - \hat{g}_3(z, q^2, m^2)\Big) = 0\,. \tag{4.30}$$

Therefore the result obtained for the Burkhardt-Cottingham sum rule is not unique. On the other hand the sum rule in (see Eq. (3.60)) is non-vanishing since it receives order α_s contributions.

$$\begin{aligned}
&\int_0^1 dz\, \Big(2\, z\, \hat{g}_5(z, q^2, m^2) - \hat{g}_3(z, q^2, m^2)\Big) = \\
&\Big(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\Big)\, \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg[\Bigg(- \frac{29q^2}{2m^2} - \frac{17q^2}{2(m^2+q^2)} + \frac{7q^4}{(m^2+q^2)^2} - \frac{4q^6}{(m^2+q^2)^3}\Bigg) \\
&\times \ln\frac{t-1}{\sqrt{t}} + \frac{1}{\sqrt{q^4 - 4m^2q^2}}\Bigg(- \frac{13q^4}{2m^2} + 12m^2 + 23q^2\Bigg)\ln(t) - 8 - \frac{3q^2}{2m^2} - \frac{5q^2}{2(m^2+q^2)} \\
&+ \frac{2q^4}{(m^2+q^2)^2} + \Bigg(\frac{3q^4}{4m^2} + \frac{23q^2}{2}\Bigg)\mathcal{J}\Bigg] \\
&\underset{-q^2 \gg m^2}{=} \Big(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\Big)\, \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg[\frac{4}{3} + \Bigg(\frac{m^2}{-q^2}\Bigg)\Bigg(\frac{1124}{225} + \frac{76}{15}\ln\Bigg(\frac{m^2}{-q^2}\Bigg)\Bigg)\Bigg]\,.
\end{aligned} \tag{4.31}$$

Finally we also present the lowest order corrections to the sum rules given in Eqs. (3.61), (3.62). The former receives higher order corrections and it reads

$$
\begin{aligned}
&(\Gamma_q^5)^{-1} \int_0^1 \frac{dz}{z} \, \Big(\hat{g}_3(z,q^2,m^2) + 2\,z\,\hat{g}_9(z,q^2,m^2)\Big) = \\
&-2\,\Big(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\Big) \Bigg[1 + \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg\{ \Bigg(-8 + \frac{2q^4}{(m^2+q^2)^2} \Bigg) \ln \frac{t-1}{\sqrt{t}} \\
&+ \frac{1}{\sqrt{q^4 - 4m^2 q^2}} \Bigg(8m^2 - 3q^2\Bigg) \ln(t) + \frac{m^2}{(m^2+q^2)} + 4m^2 \mathcal{J} \Bigg\}\Bigg] \\
&\underset{-q^2 \gg m^2}{=} -2\,\Big(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\Big) \Bigg[1 + \frac{\alpha_s(\mu^2)}{4\pi} C_F \Bigg\{ \frac{3m^2}{q^2} \Bigg\}\Bigg]\,. \qquad (4.32)
\end{aligned}
$$

On the other hand the sum rule in Eq. (3.62) does not get any corrections and it reads

$$
(\Gamma_q^5)^{-1} \int_0^1 \frac{dz}{z} \, \Big(\hat{g}_3(z,q^2,m^2) - 2\,z\,\hat{g}_8(x,q^2,m^2)\Big) = -2\,\Big(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\Big)\,. \qquad (4.33)
$$

One observes a similarity in behaviour between the sum rules in Eqs. (4.25) and Eqs. (4.33) which also holds for Eqs. (4.26) and (4.32). The reason for the vanishing of the first order contributions to the sum rules in Eqs. (4.29), (4.30), (4.33) is rather obscure. Probably it is the consequence of a super-convergence relation. Suppose the forward Compton scattering amplitude $\Delta T(\nu, q^2, m^2)$ $(\nu = p \cdot q/m)$ behaves asymptotically like ν^{-1-a} with $a > 0$ then it satisfies the unsubtracted dispersion relation

$$
\begin{aligned}
\Delta T(\nu,q^2,m^2) &= \frac{1}{\pi} \int_{-q^2/2m}^{\infty} d\nu' \, \frac{\Delta F(\nu',q^2,m^2)}{\nu' - \nu}\,, \\
\Delta F(\nu,q^2,m^2) &= \mathrm{Im}\,\Delta T(\nu,q^2,m^2)\,, \qquad (4.34)
\end{aligned}
$$

where the symbol Δ denotes a combination of amplitudes or structure functions. Taking the limit $\nu \to \infty$ provides us with the super convergence relation

$$
\int_0^1 \frac{dx}{x^2} \, \Delta F(x,q^2,m^2) = 0\,, \qquad (4.35)
$$

In the derivation of the formula above we have used the substitution $\nu = -q^2/2mx$. If we take $\Delta F = \hat{g}_2/\nu^2$ and $\Delta F = (\hat{g}_4 - \hat{g}_3)/\nu^2$, Eqs. (4.29) and (4.30) follow automatically. Apparently ΔT satisfies the requirement for the asymptotic behaviour given above. In a similar way one can take $\Delta F = (\hat{g}_3^{(1)} - 2\,x\,\hat{g}_8^{(1)})/\nu$ leading to the vanishing of the order α_s correction in Eqs. (4.33). However there is no general argument in quantum field theory that this behaviour persists in all orders. In fact the non-zero contributions in Eqs. (4.31), (4.32) already indicate that the reasons behind the existence of the super convergence relation are obscure. The only argument is given by

the ETC algebra in Eq. (2.5) which predicts that the Adler sum rule Eq. (2.24) and its polarized analogue in Eq. (2.26) do not receive any higher order corrections.

Summarizing our findings we conclude that there exist two type of sum rules which, because of their origin, are called fundamental and non-fundamental sum rules. The former are related to expectation values of conserved currents or partially conserved axial-vector currents which are sandwiched between hadronic states. The ones which originate from the equal time current (ETC) algebra are independent of the nature of the (axial-) vector currents and they do not receive higher order contributions in perturbation theory or power corrections. The other ones only hold for fermionic currents characteristic of QCD and they receive QCD as well as power corrections. Both types provide us with stringent tests for QCD. The non-fundamental sum rules which originate from the leading twist parton model are unstable against higher order QCD corrections at least from second order in α_s onwards. They receive scaling violating terms which indicates that they are heavily broken. Also non-fundamental are those sum rules which appear in polarized scattering and contain twist three contributions. One of them is the Burkhardt-Cottingham sum rule, derived for the polarized structure function g_2, which vanishes up to first order for the vector current as well as axial-vector current. However this sum rule is not unique since this property also holds for other structure functions. In quantum field theory the origin of this property is still obscure and we do not know whether the corrections to these sum rules will vanish in higher order.

ACKNOWLEDGMENTS

The work of W.L. van Neerven was supported by the EC network 'QCD and Particle Structure' under contract No. FMRX–CT98–0194.

A Appendix A

In this section we present the sum rules in the case of four flavours i.e. $SU(4)_F$. For this group the charged current in Eq. (2.16) becomes equal to

$$\begin{aligned} J^\mu_\pm(y) &= \Big(V^\mu_{1\pm i2}(y) - A^\mu_{1\pm i2}(y) + V^\mu_{13\mp i14}(y) - A^\mu_{13\mp i14}(y)\Big)\cos\theta_c \\ &\quad + \Big(V^\mu_{4\pm i5}(y) - A^\mu_{4\pm i5}(y) - V^\mu_{11\mp i12}(y) + A^\mu_{11\mp i12}(y)\Big)\sin\theta_c\,. \end{aligned} \tag{A.1}$$

Using the infinite momentum frame technique the Adler sum rule equals

$$\int_0^1 \frac{d\,x}{x}\Big(F_2^{\mathrm{W^-N}}(x,Q^2) - F_2^{\mathrm{W^+N}}(x,Q^2)\Big) = 4\,I_3^N + 2\,S^N + 2\,C^N\,, \tag{A.2}$$

and for polarized scattering we obtain

$$\begin{aligned} &\int_0^1 \frac{d\,x}{x}\Big(g_4^{\mathrm{W^-N}}(x,Q^2) - g_4^{\mathrm{W^+N}}(x,Q^2)\Big) = -4\,I_3^N(F'+D') - 2\,F' \\ &+\frac{2}{3}(D' + 2\,E)\,. \end{aligned} \tag{A.3}$$

In the formulae above the vector charges (see Eq. (2.25) are given by

$$\begin{aligned} &\Gamma_3^N = 2\,I_3^N\,, \quad \Gamma_8^N = \sqrt{3}\,(B^N + S^N - \frac{1}{3}\,C^N)\,, \quad \Gamma_{15}^N = \frac{1}{2}\sqrt{6}\,(B^N - \frac{4}{3}\,C^N)\,, \\ &\Gamma_0^N = 3\,B^N\,, \end{aligned} \tag{A.4}$$

where C^N denotes the quantum number for charm. The axial-vector charges (see Eq. (2.27)) are equal to

$$\Gamma_3^{5,N} = 2\,I_3^N(F'+D')\,, \qquad \Gamma_8^{5,N} = \frac{1}{3}\sqrt{3}\,(3\,F' - D')\,, \qquad \Gamma_{15}^{5,N} = \frac{1}{3}\sqrt{6}\,E\,. \tag{A.5}$$

Besides E, which emerges from the expectation value $\langle N(p,s)|A^\mu_{15}(0)|N(p,s)\langle$ when the flavour symmetry is given by $SU(4)$, the numerical values for F and D change from those given for $SU(3)_F$. Therefore we have indicated them by a prime because the original values were obtained by assuming a $SU(3)_F$ symmetry. In the case of the proton (p) and the neutron (n) the quantum numbers above are given by

$$\begin{aligned} &I_3^p = \frac{1}{2}\,, \qquad B^p = 1\,, \qquad S^p = 0\,, \qquad C^p = 0\,, \\ &I_3^n = -\frac{1}{2}\,, \qquad B^n = 1\,, \qquad S^n = 0 \qquad C^n = 0\,. \end{aligned} \tag{A.6}$$

Following the same procedure as in the case of three flavours one can derive from the light-cone current algebra in Eqs. (3.1), (3.2) the charged current structure functions for four flavours

$$F_2^{\mathrm{W^+N}}(x)/x = 2\,\bar{V}_0^1(x) - 2\,V_3^1(x) - \frac{2}{3}\sqrt{3}\,V_8^1(x) + \frac{2}{3}\sqrt{6}\,V_{15}^1(x)\,, \tag{A.7}$$

$$F_3^{\mathrm{W^+N}}(x)/x = 2\,V_0^1(x) - 2\,\bar{V}_3^1(x) - \frac{2}{3}\sqrt{3}\,\bar{V}_8^1(x) + \frac{2}{3}\sqrt{6}\,\bar{V}_{15}^1(x)\,, \tag{A.8}$$

$$\begin{aligned} g_1^{\mathrm{W^+N}}(x) &= \left(A_0^1(x) + \frac{\partial A_0^2(x)}{\partial x}\right) - \left(\bar{A}_3^1(x) + \frac{\partial \bar{A}_3^2(x)}{\partial x}\right) - \frac{1}{3}\sqrt{3}\,\left(\bar{A}_8^1(x) + \frac{\partial \bar{A}_8^2(x)}{\partial x}\right) \\ &\quad + \frac{1}{3}\sqrt{6}\,\left(\bar{A}_{15}^1(x) + \frac{\partial \bar{A}_{15}^2(x)}{\partial x}\right)\,, \end{aligned} \tag{A.9}$$

$$\begin{aligned} g_4^{\mathrm{W^+N}}(x)/x &= -2\,\left(\bar{A}_0^1(x) + \frac{\partial \bar{A}_0^2(x)}{\partial x}\right) + 2\,\left(A_3^1(x) + \frac{\partial A_3^2(x)}{\partial x}\right) \\ &\quad + \frac{2}{3}\sqrt{3}\,\left(A_8^1(x) + \frac{\partial A_8^2(x)}{\partial x}\right) - \frac{2}{3}\sqrt{6}\,\left(A_{15}^1(x) + \frac{\partial A_{15}^2(x)}{\partial x}\right)\,. \end{aligned} \tag{A.10}$$

The structure functions for $F_i^{\mathrm{W^-p}}$ and $g_i^{\mathrm{W^-p}}$ are obtained in the same way as mentioned below Eqs. (3.16)-(3.19). The sum rules are given by

unpolarized Bjorken sum rule

$$\int_0^1 d\,x\,\left(F_1^{\mathrm{W^-N}}(x,Q^2) - F_1^{\mathrm{W^+N}}(x,Q^2)\right) = 2\,I_3^N + S^N + C^N\,, \tag{A.11}$$

Gross Llewellyn Smith sum rule

$$\int_0^1 d\,x\,\left(F_3^{\mathrm{W^-N}}(x,Q^2) + F_3^{\mathrm{W^+N}}(x,Q^2)\right) = 6\,B^N\,, \tag{A.12}$$

$$\begin{aligned} \int_0^1 d\,x\,\left(g_5^{\mathrm{W^-N}}(x,Q^2) - g_5^{\mathrm{W^+N}}(x,Q^2)\right) &= -2\,I_3^N\,(F' + D') - F' \\ &\quad + \frac{1}{3}\,(D' + 2\,E)\,. \end{aligned} \tag{A.13}$$

The neutral current structure functions are given by the same expressions as derived for $SU(3)_F$ in Eqs. (3.34)-(3.43) except that one has to replace the following equations. The neutral currents in Eq. (3.29) are now given by

$$V_\gamma^\mu(y) \equiv V_3^\mu(y) + \frac{1}{3}\sqrt{3}\,V_8^\mu(y) - \frac{1}{3}\sqrt{6}\,V_{15}^\mu(y) + \frac{1}{3}\,V_0^\mu(y)\,,$$

$$A^{\mu}_{\gamma}(y) \equiv A^{\mu}_{3}(y) + \frac{1}{3}\sqrt{3}\, A^{\mu}_{8}(y) - \frac{1}{3}\sqrt{6}\, A^{\mu}_{15}(y) + \frac{1}{3}\, A^{\mu}_{0}(y)\,, \qquad \text{(A.14)}$$

and the formulae in Eq. (3.33) are replaced by

$$\begin{aligned}
V_{\gamma}(x) &= V^1_3(x) + \frac{1}{3}\sqrt{3}V^1_8(x) - \frac{1}{3}\sqrt{6}V^1_8(x) + \frac{1}{3}V^1_0(x)\,,\\
A_{\gamma}(x) &= A^1_3(x) + \frac{\partial A^2_3(x)}{\partial x} + \frac{1}{3}\sqrt{3}\left(A^1_8(x) + \frac{\partial A^2_8(x)}{\partial x}\right) - \frac{1}{3}\sqrt{6}\left(A^1_{15}(x) + \frac{\partial A^2_{15}(x)}{\partial x}\right)\\
&\quad + \frac{1}{3}\left(A^1_0(x) + \frac{\partial A^2_0(x)}{\partial x}\right)\,,\\
V_{\gamma\gamma}(x) &= \frac{5}{9}\,V^1_0(x) + \frac{1}{3}\,V^1_3(x) + \frac{1}{9}\sqrt{3}\,V^1_8(x) - \frac{1}{9}\sqrt{6}\,V^1_{15}(x)\,,\\
A_{\gamma\gamma}(x) &= \frac{5}{9}\left(A^1_0(x) + \frac{\partial A^2_0(x)}{\partial x}\right) + \frac{1}{3}\left(A^1_3(x) + \frac{\partial A^2_3(x)}{\partial x}\right)\\
&\quad + \frac{1}{9}\sqrt{3}\left(A^1_8(x) + \frac{\partial A^2_8(x)}{\partial x}\right) - \frac{1}{9}\sqrt{6}\left(A^1_{15}(x) + \frac{\partial A^2_{15}(x)}{\partial x}\right)\,. \qquad \text{(A.15)}
\end{aligned}$$

The sum rules in Eqs. (3.44), (3.45) become for $SU(4)_F$

$$\int_0^1 dx\, F_3^{\mathrm{ZN}}(x,Q^2) = \frac{3}{2}\,(1-2\,s^2)\,B^N - \frac{1}{3}\,s^2\,(2\,I_3^N + S^N + C^N)\,, \qquad \text{(A.16)}$$

$$\int_0^1 dx\, F_3^{\gamma\mathrm{Z\ N}}(x,Q^2) = \frac{3}{2}\,B^N + + \frac{1}{6}\,(2\,I_3^N + S^N + C^N)\,. \qquad \text{(A.17)}$$

The sum rules for the longitudinal structure function g_1 have the same form as in Eqs. (3.46)-(3.48). However the shorthand notations in Eq. (3.49) read

$$\begin{aligned}
f^{\mathrm{NS}} &= \frac{1}{6}\,I_3^N\,(F'+D') + \frac{1}{12}\,F' - \frac{1}{36}(D' + 2\,E)\,,\\
f^{\mathrm{S}}_{\gamma} &= \frac{5}{18}\,\Gamma_0^{5,N}\,,\\
f^{\mathrm{S}}_{\mathrm{Z}} &= (\frac{1}{2} - s^2 + \frac{10}{9}\,s^4)\,\Gamma_0^{5,N}\,,\\
f^{\mathrm{S}}_{\gamma Z} &= (\frac{1}{2} - \frac{10}{9}\,s^2)\,\Gamma_0^{5,N}\,. \qquad \text{(A.18)}
\end{aligned}$$

The bilocal operator matrix elements are expressed into the parton densities in the same way as presented in Eqs. (3.54), (3.56) except that one has additional matrix

elements due to the generator λ_{15} appearing in $SU(4)$. The latter are given by

$$
\begin{aligned}
V_{15}^{1}(x) &= \frac{1}{2\sqrt{6}}\Big(V_u(x)+V_d(x)+V_s(x)-3\,V_c(x)\Big)\,, \\
\bar{V}_{15}^{1}(x) &= \frac{1}{2\sqrt{6}}\Big(\Delta_u(x)+\Delta_d(x)+\Delta_s(x)-3\,\Delta_c(x)\Big)\,, \\
A_{15}^{1}(x)+\frac{\partial A_{15}^{2}(x)}{\partial x} &= \frac{1}{2\sqrt{6}}\Big(\Delta_{\delta u}(x)+\Delta_{\delta d}(x)+\Delta_{\delta s}(x)-3\,\Delta_{\delta c}(x)\Big)\,, \\
\bar{A}_{15}^{1}(x)+\frac{\partial \bar{A}_{15}^{2}(x)}{\partial x} &= \frac{1}{2\sqrt{6}}\Big(V_{\delta u}(x)+V_{\delta d}(x)+V_{\delta s}(x)-3\,V_{\delta c}(x)\Big)\,. \qquad \text{(A.19)}
\end{aligned}
$$

The definitions for the non-singlet and singlet quark densities are changed into

$$
\begin{aligned}
&V_q(x)=q(x)-\bar{q}(x)\,, \qquad \Delta_q(x)=q(x)+\bar{q}(x)-\frac{1}{4}\Sigma(x)\,, \\
&\Sigma(x)=\sum_{q=u,d,s,c} q(x)+\bar{q}(x)\,, \qquad \text{(A.20)}
\end{aligned}
$$

with similar notations for the polarized quark densities.

B Appendix B

In this appendix we only present the results for the structure functions which are observed in the cross section when the lepton masses are neglected. Therefore we omit the results for F_i $(i = 4, 5)$ and g_i $(i = 6-9)$. The order α_s contributions to the integral in Eq. (4.20), coming from the sum of soft and virtual gluon corrections, are given by

$$\mathcal{T}_{1,q}^{\mathrm{S+V}}(z) = C_F\left[v_q^{V_1} v_q^{V_2}\mathcal{R}^{S+V} + a_q^{V_1} a_q^{V_2}\left(1-\frac{4m^2}{q^2}\right)\left(\mathcal{R}^{S+V} + 2\mathcal{R}_2\right)\right], \tag{B.1}$$

$$\mathcal{T}_{2,q}^{\mathrm{S+V}}(z) = 2C_F\left[v_q^{V_1} v_q^{V_2}\left(\mathcal{R}^{S+V} + \mathcal{R}_2\right) + a_q^{V_1} a_q^{V_2}\left(\mathcal{R}^{S+V} + 2\mathcal{R}_2\right)\right], \tag{B.2}$$

$$\mathcal{T}_{3,q}^{\mathrm{S+V}}(z) = 2C_F\left[\left(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\right)\left(\mathcal{R}^{S+V} + \mathcal{R}_2\right)\right], \tag{B.3}$$

$$\delta\mathcal{T}_{1,q}^{\mathrm{S+V}}(z) = C_F\left[v_q^{V_1} v_q^{V_2}\left(\mathcal{R}^{S+V} + \frac{1}{2}\mathcal{R}_2\right) + a_q^{V_1} a_q^{V_2}\left(\mathcal{R}^{S+V} + 2\mathcal{R}_2\right)\right], \tag{B.4}$$

$$\delta\mathcal{T}_{2,q}^{\mathrm{S+V}}(z) = -\frac{q^2}{8m^2}C_F\left[v_q^{V_1} v_q^{V_2}\mathcal{R}_2 + a_q^{V_1} a_q^{V_2}\mathcal{R}_3\right], \tag{B.5}$$

$$\delta\mathcal{T}_{3,q}^{\mathrm{S+V}}(z) = -C_F\left[\left(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\right)\left(2\mathcal{R}^{S+V} + 3\mathcal{R}_2 - \frac{q^2}{4m^2}\mathcal{R}_2\right)\right], \tag{B.6}$$

$$\delta\mathcal{T}_{4,q}^{\mathrm{S+V}}(z) = -C_F\left[\left(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\right)\left(2\mathcal{R}^{S+V} + 3\mathcal{R}_2\right)\right], \tag{B.7}$$

$$\delta\mathcal{T}_{5,q}^{\mathrm{S+V}}(z) = -C_F\left[\left(v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}\right)\left(\mathcal{R}^{S+V} + \mathcal{R}_2\right)\right], \tag{B.8}$$

with the definition

$$\begin{aligned}\mathcal{R}^{S+V} \;=\;& S^{\mathrm{SOFT}} + \mathcal{R}_1 = \frac{\alpha_s}{4\pi}C_F\Bigg[\left(\frac{4m^2-2q^2}{\sqrt{q^4-4m^2q^2}}\ln(t) - 2\right)\ln\left(\frac{4\omega^2}{m^2}\right)\\ &+\frac{4m^2-2q^2}{\sqrt{q^4-4m^2q^2}}\Bigg\{\frac{9}{2}\ln^2(t) - 4\ln(t)\ln(t+1) - 2\ln(t)\ln(t-1)\\ &+2\mathcal{L}i_2\left(\frac{1}{t}\right) + 4\mathcal{L}i_2\left(-\frac{1}{t}\right)\Bigg\} - 2 + \frac{16m^2-5q^2}{\sqrt{q^4-4m^2q^2}}\ln(t)\Bigg].\end{aligned} \tag{B.9}$$

We next present the results of the hard gluon contribution to the partonic structure functions indicated by $\mathcal{F}_{i,q}^{\rm HARD}$ in Eq. (4.12). The results are expressed in terms of the centre of mass energy squared of the incoming quark and vector boson in Eq. (4.11) which is denoted by $\hat{s} = (p+q)^2$. The expressions for the partonic structure functions are written in such a way that they all start with a common factor $(\hat{s}-m^2-q^2)^2/q^2$ which is cancelled if the integration variable z in Eq. (4.12) is replaced by $z = -q^2/(\hat{s}-m^2-q^2)$ i.e.

$$\int_0^{z_{max}} dz\, \hat{\mathcal{F}}_{i,q}^{\rm HARD}(z,q^2,m^2) = \int_{m^2+2m\omega}^{\infty} d\hat{s}\, \frac{-q^2}{(\hat{s}-m^2-q^2)^2}\, \hat{\mathcal{F}}_{i,q}^{\rm HARD}(\hat{s},q^2,m^2)\,. \qquad \text{(B.10)}$$

Further we introduce the shorthand notation

$$\xi = \frac{\hat{s}+m^2-q^2-\sqrt{\lambda}}{\hat{s}+m^2-q^2+\sqrt{\lambda}}\,, \quad \text{with} \quad \lambda = (\hat{s}-m^2-q^2)^2 - 4m^2q^2\,. \qquad \text{(B.11)}$$

The results for $\mathcal{F}_{i,q}^{\rm HARD}$ are given by

$$\begin{aligned}
\hat{\mathcal{F}}_{1,q}^{\rm HARD} = {}& v_q^{V_1} v_q^{V_2} C_F \frac{(\hat{s}-m^2-q^2)^2}{q^2}\Bigg[\Bigg\{(4m^2-2q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)} + \Bigg(-2m^4+\frac{7m^2q^2}{2} \\
& -q^4\Bigg)\frac{1}{\lambda^{\frac{3}{2}}} + \left(2m^2+\frac{q^2}{2}\right)\frac{\hat{s}}{\lambda^{\frac{3}{2}}} + \left(2m^2q^2-\frac{q^4}{2}\right)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)^2} \\
& +\left(-6m^2+\frac{q^2}{2}\right)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Bigg\}\ln\xi + \Bigg(-\frac{2q^6}{(m^2+q^2)^3} \\
& +\frac{3q^4}{(m^2+q^2)^2} - \frac{5q^2}{4(m^2+q^2)} + \frac{q^2}{4(m^2-q^2)}\Bigg)\frac{1}{\hat{s}} + \Bigg(-\frac{q^6}{(m^2+q^2)^2} \\
& +\frac{3q^4}{2(m^2+q^2)} - \frac{q^2}{2}\Bigg)\frac{1}{\hat{s}^2} + \Bigg(-2m^2+\frac{q^4}{4m^2}+\frac{q^4}{(m^2-q^2)}+\frac{q^2}{2}\Bigg)\frac{1}{\lambda} \\
& +\left(2-\frac{q^2}{4m^2}-\frac{q^2}{4(m^2-q^2)}\right)\frac{\hat{s}}{\lambda} + \frac{4}{(\hat{s}-m^2)} + \Bigg(-\frac{q^6}{(m^2+q^2)^2} \\
& +\frac{3q^4}{2(m^2+q^2)} + 2q^2\Bigg)\frac{1}{(\hat{s}-m^2-q^2)^2} + \Bigg(-6+\frac{q^2}{4m^2}+\frac{2q^6}{(m^2+q^2)^3} \\
& -\frac{3q^4}{(m^2+q^2)^2} + \frac{5q^2}{4(m^2+q^2)}\Bigg)\frac{1}{(\hat{s}-m^2-q^2)}\Bigg] \\
& +a_q^{V_1} a_q^{V_2} C_F \frac{(\hat{s}-m^2-q^2)^2}{q^2}\Bigg[\Bigg\{\left(-\frac{16m^4}{q^2}+12m^2-2q^2\right)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}
\end{aligned}$$

$$+\Bigg(-4m^4+\frac{7m^2q^2}{2}-q^4\Bigg)\frac{1}{\lambda^{\frac{3}{2}}}+\Bigg(4m^2+\frac{q^2}{2}\Bigg)\frac{\hat{s}}{\lambda^{\frac{3}{2}}}-\Bigg(16m^4+\frac{q^4}{2}\Bigg)$$

$$\times\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)^2}+(\frac{16m^4}{q^2}-16m^2+\frac{q^2}{2}\Bigg)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Bigg\}\ln\xi$$

$$+\Bigg(-\frac{2q^6}{(m^2+q^2)^3}+\frac{5q^4}{(m^2+q^2)^2}-\frac{17q^2}{4(m^2+q^2)}-\frac{3q^2}{4(m^2-q^2)}\Bigg)\frac{1}{\hat{s}}$$

$$+\Bigg(-\frac{q^6}{(m^2+q^2)^2}+\frac{3q^4}{2(m^2+q^2)}-\frac{q^2}{2}\Bigg)\frac{1}{\hat{s}^2}+\Bigg(-4m^2+\frac{q^4}{4m^2}-\frac{3q^4}{(m^2-q^2)}$$

$$-\frac{9q^2}{2}\Bigg)\frac{1}{\lambda}+\Bigg(4-\frac{q^2}{4m^2}+\frac{3q^2}{4(m^2-q^2)}\Bigg)\frac{\hat{s}}{\lambda}+\Bigg(4-\frac{16m^2}{q^2}\Bigg)\frac{1}{(\hat{s}-m^2)}$$

$$+\Bigg(-16m^2-\frac{q^6}{(m^2+q^2)^2}+\frac{7q^4}{2(m^2+q^2)}\Bigg)\frac{1}{(\hat{s}-m^2-q^2)^2}+\Bigg(-8+\frac{16m^2}{q^2}$$

$$+\frac{q^2}{4m^2}+\frac{2q^6}{(m^2+q^2)^3}-\frac{5q^4}{(m^2+q^2)^2}+\frac{17q^2}{4(m^2+q^2)}\Bigg)\frac{1}{(\hat{s}-m^2-q^2)}\Bigg], \quad \text{(B.12)}$$

$$\hat{\mathcal{F}}_{2,q}^{\rm HARD} = v_q^{V_1}v_q^{V_2}C_F\frac{(\hat{s}-m^2-q^2)^2}{q^2}\Bigg[\Bigg\{(8m^2-4q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}+\Bigg(3m^2q^2+\frac{q^6}{4m^2}$$

$$-\frac{27q^4}{4}\Bigg)\frac{1}{\lambda^{\frac{3}{2}}}+\Bigg(-12m^4q^4+21m^2q^6-3q^8\Bigg)\frac{1}{\lambda^{\frac{5}{2}}}+\Bigg(-\frac{q^4}{4m^2}+5q^2\Bigg)\frac{\hat{s}}{\lambda^{\frac{3}{2}}}$$

$$+(-36m^2q^4+3q^6)\frac{\hat{s}}{\lambda^{\frac{5}{2}}}+\Bigg(-8m^2+\frac{q^4}{4m^2}-q^2\Bigg)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Bigg\}\ln\xi$$

$$+\Bigg(\frac{q^6}{(m^2-q^2)^3}-\frac{q^4}{2(m^2+q^2)^2}-\frac{q^4}{(m^2-q^2)^2}+\frac{3q^2}{4(m^2+q^2)}$$

$$-\frac{3q^2}{4(m^2-q^2)}\Bigg)\frac{1}{\hat{s}}+\Bigg(-\frac{q^4}{2(m^2+q^2)}-\frac{q^4}{2(m^2-q^2)}\Bigg)\frac{1}{\hat{s}^2}$$

$$+\Bigg(-\frac{q^4}{4m^2}+\frac{4q^8}{(m^2-q^2)^3}-\frac{2q^6}{(m^2-q^2)^2}-\frac{9q^4}{2(m^2-q^2)}+7q^2\Bigg)\frac{1}{\lambda}$$

$$+\Bigg(-36m^2q^4+\frac{12q^8}{(m^2-q^2)}+27q^6\Bigg)\frac{1}{\lambda^2}+\Bigg(\frac{q^2}{4m^2}-\frac{q^6}{(m^2-q^2)^3}$$

$$+\frac{q^4}{(m^2-q^2)^2}+\frac{3q^2}{4(m^2-q^2)}\Big)\frac{\hat{s}}{\lambda}+\Big(-\frac{3q^6}{(m^2-q^2)}-12q^4\Big)\frac{\hat{s}}{\lambda^2}+\frac{8}{(\hat{s}-m^2)}$$

$$+\Big(-8-\frac{q^2}{4m^2}+\frac{q^4}{2(m^2+q^2)^2}-\frac{3q^2}{4(m^2+q^2)}\Big)\frac{1}{(\hat{s}-m^2-q^2)}\Big]$$

$$+a_q^{V_1}a_q^{V_2}C_F\frac{(\hat{s}-m^2-q^2)^2}{q^2}\Big[\Big\{(8m^2-4q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}-\Big(6m^2q^2-\frac{q^6}{4m^2}$$

$$+\frac{31q^4}{4}\Big)\frac{1}{\lambda^{\frac{3}{2}}}+\Big(-48m^4q^4+33m^2q^6-3q^8\Big)\frac{1}{\lambda^{\frac{5}{2}}}+\Big(-\frac{q^4}{4m^2}+6q^2\Big)\frac{\hat{s}}{\lambda^{\frac{3}{2}}}$$

$$+(-48m^2q^4+3q^6)\frac{\hat{s}}{\lambda^{\frac{5}{2}}}+\Big(-8m^2+\frac{q^4}{4m^2}-2q^2\Big)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Big\}\ln\xi$$

$$+\Big(-\frac{3q^6}{(m^2-q^2)^3}-\frac{5q^4}{2(m^2+q^2)^2}-\frac{9q^4}{(m^2-q^2)^2}+\frac{7q^2}{4(m^2+q^2)}$$

$$-\frac{23q^2}{4(m^2-q^2)}\Big)\frac{1}{\hat{s}}+\Big(-\frac{5q^4}{2(m^2+q^2)}+\frac{3q^4}{2(m^2-q^2)}+4q^2\Big)\frac{1}{\hat{s}^2}$$

$$+\Big(-\frac{q^4}{4m^2}-\frac{12q^8}{(m^2-q^2)^3}-\frac{42q^6}{(m^2-q^2)^2}-\frac{85q^4}{2(m^2-q^2)}-6q^2\Big)\frac{1}{\lambda}$$

$$+\Big(-96m^2q^4-\frac{36q^8}{(m^2-q^2)}-21q^6\Big)\frac{1}{\lambda^2}+\Big(\frac{q^2}{4m^2}+\frac{3q^6}{(m^2-q^2)^3}$$

$$+\frac{9q^4}{(m^2-q^2)^2}+\frac{23q^2}{4(m^2-q^2)}\Big)\frac{\hat{s}}{\lambda}+\frac{9q^6}{(m^2-q^2)}\frac{\hat{s}}{\lambda^2}+\frac{8}{(\hat{s}-m^2)}$$

$$+\Big(-8-\frac{q^2}{4m^2}+\frac{5q^4}{2(m^2+q^2)^2}-\frac{7q^2}{4(m^2+q^2)}\Big)\frac{1}{(\hat{s}-m^2-q^2)}\Big], \tag{B.13}$$

$$\hat{\mathcal{F}}_{3,q}^{\mathrm{HARD}}=\Big(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\Big)C_F\frac{(\hat{s}-m^2-q^2)^2}{2\,q^2}\Big[\Big\{(16m^2-8q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}$$

$$+(12m^2q^2-8q^4)\frac{1}{\lambda^{\frac{3}{2}}}+4q^2\frac{\hat{s}}{\lambda^{\frac{3}{2}}}-\frac{16m^2}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Big\}\ln\xi+\Big(-\frac{2q^4}{(m^2+q^2)^2}$$

$$+\frac{2q^2}{(m^2+q^2)}+\frac{2q^2}{(m^2-q^2)}\Big)\frac{1}{\hat{s}}+\Big(-\frac{2q^4}{(m^2+q^2)}+2q^2\Big)\frac{1}{\hat{s}^2}+\Big(\frac{2q^4}{m^2}$$

$$+\frac{8q^4}{(m^2-q^2)}+20q^2\Bigg)\frac{1}{\lambda}+\Bigg(-\frac{2q^2}{m^2}-\frac{2q^2}{(m^2-q^2)}\Bigg)\frac{\hat{s}}{\lambda}+\frac{16}{(\hat{s}-m^2)}$$

$$+\Bigg(-16+\frac{2q^2}{m^2}+\frac{2q^4}{(m^2+q^2)^2}-\frac{2q^2}{(m^2+q^2)}\Bigg)\frac{1}{(\hat{s}-m^2-q^2)}\Bigg]. \tag{B.14}$$

The computation of the polarized structure functions proceeds in a similar way. We obtain

$$\hat{g}_{1,q}^{\rm HARD} = v_q^{V_1}v_q^{V_2}C_F\frac{(\hat{s}-m^2-q^2)^2}{q^2}\Bigg[\Bigg\{(4m^2-2q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}+\Bigg(\frac{3m^2q^2}{4}-\frac{9q^4}{4}\Bigg)\frac{1}{\lambda^{\frac{3}{2}}}$$

$$+(3m^4q^4+3m^2q^6)\frac{1}{\lambda^{\frac{5}{2}}}+\frac{5q^2}{4}\frac{\hat{s}}{\lambda^{\frac{3}{2}}}-15m^2q^4\frac{\hat{s}}{\lambda^{\frac{5}{2}}}$$

$$+\Bigg(-4m^2-\frac{q^2}{4}\Bigg)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Bigg\}\ln\xi+\Bigg(\frac{q^6}{(m^2-q^2)^3}+\frac{q^4}{(m^2-q^2)^2}$$

$$-\frac{q^2}{4(m^2+q^2)}+\frac{q^2}{4(m^2-q^2)}\Bigg)\frac{1}{\hat{s}}+\Bigg(-\frac{q^4}{2(m^2-q^2)}-\frac{q^2}{2}\Bigg)\frac{1}{\hat{s}^2}+\Bigg(-\frac{q^4}{2m^2}$$

$$+\frac{4q^8}{(m^2-q^2)^3}+\frac{6q^6}{(m^2-q^2)^2}+\frac{7q^4}{2(m^2-q^2)}+\frac{7q^2}{4}\Bigg)\frac{1}{\lambda}$$

$$+\Bigg(-3m^2q^4+\frac{12q^8}{(m^2-q^2)}+18q^6\Bigg)\frac{1}{\lambda^2}+\Bigg(\frac{q^2}{2m^2}-\frac{q^6}{(m^2-q^2)^3}$$

$$-\frac{q^4}{(m^2-q^2)^2}-\frac{q^2}{4(m^2-q^2)}\Bigg)\frac{\hat{s}}{\lambda}+\Bigg(-\frac{3q^6}{(m^2-q^2)}-9q^4\Bigg)\frac{\hat{s}}{\lambda^2}+\frac{4}{(\hat{s}-m^2)}$$

$$+\Bigg(-4-\frac{q^2}{2m^2}+\frac{q^2}{4(m^2+q^2)}\Bigg)\frac{1}{(\hat{s}-m^2-q^2)}\Bigg]$$

$$+a_q^{V_1}a_q^{V_2}C_F\frac{(\hat{s}-m^2-q^2)^2}{q^2}\Bigg[\Bigg\{(4m^2-2q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}+\Bigg(4m^4-\frac{13m^2q^2}{4}$$

$$-\frac{9q^4}{4}\Bigg)\frac{1}{\lambda^{\frac{3}{2}}}+(-12m^6q^2+3m^4q^4+3m^2q^6)\frac{1}{\lambda^{\frac{5}{2}}}+\Bigg(-4m^2+\frac{5q^2}{4}\Bigg)\frac{\hat{s}}{\lambda^{\frac{3}{2}}}$$

$$+\Bigg(12m^4q^2-15m^2q^4\Bigg)\frac{\hat{s}}{\lambda^{\frac{5}{2}}}-\frac{q^2}{4}\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Bigg\}\ln\xi$$

$$
\begin{aligned}
&+\Big(-\frac{q^6}{(m^2-q^2)^3}-\frac{q^4}{(m^2+q^2)^2}-\frac{4q^4}{(m^2-q^2)^2}-\frac{q^2}{4(m^2+q^2)}\\
&-\frac{15q^2}{4(m^2-q^2)}\Big)\frac{1}{\hat{s}}+\Big(-\frac{q^4}{(m^2+q^2)}+\frac{q^4}{2(m^2-q^2)}+\frac{3q^2}{2}\Big)\frac{1}{\hat{s}^2}\\
&+\Big(4m^2-\frac{q^4}{2m^2}-\frac{4q^8}{(m^2-q^2)^3}-\frac{18q^6}{(m^2-q^2)^2}-\frac{47q^4}{2(m^2-q^2)}-\frac{17q^2}{4}\Big)\frac{1}{\lambda}\\
&-\Big(12m^4q^2+33m^2q^4+\frac{12q^8}{(m^2-q^2)}+6q^6\Big)\frac{1}{\lambda^2}-\Big(4-\frac{q^2}{2m^2}-\frac{q^6}{(m^2-q^2)^3}\\
&-\frac{4q^4}{(m^2-q^2)^2}-\frac{15q^2}{4(m^2-q^2)}\Big)\frac{\hat{s}}{\lambda}+\Big(12m^2q^2+\frac{3q^6}{(m^2-q^2)}-3q^4\Big)\frac{\hat{s}}{\lambda^2}\\
&+\Big(-\frac{q^2}{2m^2}+\frac{q^4}{(m^2+q^2)^2}+\frac{q^2}{4(m^2+q^2)}\Big)\frac{1}{(\hat{s}-m^2-q^2)}\\
&+\frac{4}{(\hat{s}-m^2)}\Big],
\end{aligned}
\tag{B.15}
$$

$$
\begin{aligned}
\hat{g}_{2,q}^{\rm HARD} = {}& v_q^{V_1}v_q^{V_2}C_F\frac{(\hat{s}-m^2-q^2)^2}{q^2}\Big[\Big\{(-m^2q^2+2q^4)\frac{1}{\lambda^{\frac{3}{2}}}-(3m^4q^4+3m^2q^6)\frac{1}{\lambda^{\frac{5}{2}}}\\
&+15m^2q^4\frac{\hat{s}}{\lambda^{\frac{5}{2}}}\Big\}\ln\xi-\Big(\frac{q^6}{(m^2-q^2)^3}+\frac{q^4}{(m^2-q^2)^2}+\frac{q^2}{2(m^2-q^2)}\Big)\frac{1}{\hat{s}}\\
&+\Big(\frac{q^4}{2(m^2-q^2)}+\frac{q^2}{2}\Big)\frac{1}{\hat{s}^2}-\Big(\frac{4q^8}{(m^2-q^2)^3}+\frac{6q^6}{(m^2-q^2)^2}+\frac{9q^4}{2(m^2-q^2)}\\
&+\frac{3q^2}{2}\Big)\frac{1}{\lambda}+\Big(3m^2q^4-\frac{12q^8}{(m^2-q^2)}-18q^6\Big)\frac{1}{\lambda^2}+\Big(\frac{q^6}{(m^2-q^2)^3}\\
&+\frac{q^4}{(m^2-q^2)^2}+\frac{q^2}{2(m^2-q^2)}\Big)\frac{\hat{s}}{\lambda}+\Big(\frac{3q^6}{(m^2-q^2)}+9q^4\Big)\frac{\hat{s}}{\lambda^2}\Big]\\
&+a_q^{V_1}a_q^{V_2}C_F\frac{(\hat{s}-m^2-q^2)^2}{q^2}\Big[\Big\{(-m^2q^2+2q^4)\frac{1}{\lambda^{\frac{3}{2}}}+(12m^6q^2-3m^4q^4\\
&-3m^2q^6)\frac{1}{\lambda^{\frac{5}{2}}}+(-12m^4q^2+15m^2q^4)\frac{\hat{s}}{\lambda^{\frac{5}{2}}}\Big\}\ln\xi+\Big(\frac{q^6}{(m^2-q^2)^3}+\frac{4q^4}{(m^2-q^2)^2}
\end{aligned}
$$

$$+\frac{7q^2}{2(m^2-q^2)}\Big)\frac{1}{\hat{s}}-\Big(\frac{q^4}{2(m^2-q^2)}+\frac{q^2}{2}\Big)\frac{1}{\hat{s}^2}+\Big(\frac{4q^8}{(m^2-q^2)^3}$$

$$+\frac{18q^6}{(m^2-q^2)^2}+\frac{45q^4}{2(m^2-q^2)}+\frac{17q^2}{2}\Big)\frac{1}{\lambda}+\Big(12m^4q^2+33m^2q^4$$

$$+\frac{12q^8}{(m^2-q^2)}+6q^6\Big)\frac{1}{\lambda^2}-\Big(\frac{q^6}{(m^2-q^2)^3}+\frac{4q^4}{(m^2-q^2)^2}+\frac{7q^2}{2(m^2-q^2)}\Big)\frac{\hat{s}}{\lambda}$$

$$-\Big(12m^2q^2+\frac{3q^6}{(m^2-q^2)}-3q^4\Big)\frac{\hat{s}}{\lambda^2}\Big], \tag{B.16}$$

$$\hat{g}_{3,q}^{\mathrm{HARD}}=\big(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\big)C_F\frac{(\hat{s}-m^2-q^2)^2}{2\,q^2}\Big[\Big\{(-16m^2+8q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}$$

$$+\Big(m^2q^2+\frac{q^6}{m^2}+22q^4\Big)\frac{1}{\lambda^{\frac{3}{2}}}+\big(12m^2q^6-12q^8\big)\frac{1}{\lambda^{\frac{5}{2}}}+\Big(-\frac{q^4}{m^2}-13q^2\Big)\frac{\hat{s}}{\lambda^{\frac{3}{2}}}$$

$$+\big(96m^2q^4+12q^6\big)\frac{\hat{s}}{\lambda^{\frac{5}{2}}}+\Big(16m^2+\frac{q^4}{m^2}+5q^2\Big)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Big\}\ln\xi$$

$$+\Big(\frac{2q^4}{(m^2+q^2)^2}+\frac{2q^4}{(m^2-q^2)^2}+\frac{2q^2}{(m^2-q^2)}\Big)\frac{1}{\hat{s}}+\Big(\frac{2q^4}{(m^2+q^2)}-2q^2\Big)\frac{1}{\hat{s}^2}$$

$$+\Big(\frac{5q^4}{m^2}+\frac{8q^6}{(m^2-q^2)^2}+\frac{12q^4}{(m^2-q^2)}-5q^2\Big)\frac{1}{\lambda}+(48m^2q^4-24q^6)\frac{1}{\lambda^2}$$

$$+\Big(-\frac{5q^2}{m^2}-\frac{2q^4}{(m^2-q^2)^2}-\frac{2q^2}{(m^2-q^2)}\Big)\frac{\hat{s}}{\lambda}+48q^4\frac{\hat{s}}{\lambda^2}$$

$$-\frac{16}{(\hat{s}-m^2)}+\Big(16+\frac{5q^2}{m^2}-\frac{2q^4}{(m^2+q^2)^2}\Big)\frac{1}{(\hat{s}-m^2-q^2)}\Big], \tag{B.17}$$

$$\hat{g}_{4,q}^{\mathrm{HARD}}=\big(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\big)C_F\frac{(\hat{s}-m^2-q^2)^2}{2\,q^2}\Big[\Big\{(-16m^2+8q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}$$

$$+\Big(-3m^2q^2+\frac{q^6}{m^2}+18q^4\Big)\frac{1}{\lambda^{\frac{3}{2}}}+(-108m^4q^4-156m^2q^6)\frac{1}{\lambda^{\frac{5}{2}}}$$

$$+\Big(-\frac{q^4}{m^2}-13q^2\Big)\frac{\hat{s}}{\lambda^{\frac{3}{2}}}+12m^2q^4\frac{\hat{s}}{\lambda^{\frac{5}{2}}}+\big(-120m^4q^8+120m^2q^{10}\big)\frac{1}{\lambda^{\frac{7}{2}}}$$

$$
+\Big(-960m^4q^6-120m^2q^8\Big)\frac{\hat{s}}{\lambda^{\frac{7}{2}}}+\left(16m^2+\frac{q^4}{m^2}+5q^2\right)
$$

$$
\times\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Bigg\}\ln\xi+\left(-\frac{12q^8}{(m^2-q^2)^4}-\frac{32q^6}{(m^2-q^2)^3}+\frac{2q^4}{(m^2+q^2)^2}\right.
$$

$$
\left.-\frac{20q^4}{(m^2-q^2)^2}\right)\frac{1}{\hat{s}}+\left(\frac{2q^6}{(m^2-q^2)^2}+\frac{2q^4}{(m^2+q^2)}+\frac{4q^4}{(m^2-q^2)}\right)\frac{1}{\hat{s}^2}+\left(\frac{5q^4}{m^2}\right.
$$

$$
\left.-\frac{48q^{10}}{(m^2-q^2)^4}-\frac{152q^8}{(m^2-q^2)^3}-\frac{146q^6}{(m^2-q^2)^2}-\frac{44q^4}{(m^2-q^2)}-11q^2\right)\frac{1}{\lambda}
$$

$$
+\left(-156m^2q^4-\frac{80q^{10}}{(m^2-q^2)^2}-\frac{248q^8}{(m^2-q^2)}-216q^6\right)\frac{1}{\lambda^2}+\left(-\frac{5q^2}{m^2}\right.
$$

$$
\left.+\frac{12q^8}{(m^2-q^2)^4}+\frac{32q^6}{(m^2-q^2)^3}+\frac{20q^4}{(m^2-q^2)^2}\right)\frac{\hat{s}}{\lambda}+\left(\frac{20q^8}{(m^2-q^2)^2}+\frac{52q^6}{(m^2-q^2)}\right.
$$

$$
\left.+60q^4\right)\frac{\hat{s}}{\lambda^2}-\frac{16}{(\hat{s}-m^2)}+\Big(-480m^4q^6+240m^2q^8\Big)\frac{1}{\lambda^3}-480m^2q^6\frac{\hat{s}}{\lambda^3}
$$

$$
+\left(16+\frac{5q^2}{m^2}-\frac{2q^4}{(m^2+q^2)^2}\right)\frac{1}{(\hat{s}-m^2-q^2)}\Bigg], \tag{B.18}
$$

$$
\hat{g}_{5,q}^{\mathrm{HARD}}=\Big(v_q^{V_1}a_q^{V_2}+a_q^{V_1}v_q^{V_2}\Big)C_F\frac{(\hat{s}-m^2-q^2)^2}{2\,q^2}\Bigg[\Bigg\{(-8m^2+4q^2)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2)}+\left(\frac{15m^2q^2}{2}\right.
$$

$$
\left.+\frac{11q^4}{2}\right)\frac{1}{\lambda^{\frac{3}{2}}}+(-24m^6q^2+42m^4q^4-18m^2q^6)\frac{1}{\lambda^{\frac{5}{2}}}-\frac{7q^2}{2}\frac{\hat{s}}{\lambda^{\frac{3}{2}}}+(24m^4q^2
$$

$$
+30m^2q^4)\frac{\hat{s}}{\lambda^{\frac{5}{2}}}+\left(8m^2+\frac{3q^2}{2}\right)\frac{1}{\lambda^{\frac{1}{2}}(\hat{s}-m^2-q^2)}\Bigg\}\ln\xi+\left(\frac{q^4}{(m^2+q^2)^2}\right.
$$

$$
\left.-\frac{q^4}{(m^2-q^2)^2}-\frac{3q^2}{2(m^2+q^2)}-\frac{5q^2}{2(m^2-q^2)}\right)\frac{1}{\hat{s}}+\left(\frac{q^4}{(m^2+q^2)}-q^2\right)\frac{1}{\hat{s}^2}
$$

$$
+\left(-\frac{4q^6}{(m^2-q^2)^2}-\frac{12q^4}{(m^2-q^2)}-\frac{21q^2}{2}\right)\frac{1}{\lambda}+(-24m^4q^2+18m^2q^4
$$

$$
-6q^6)\frac{1}{\lambda^2}+\left(\frac{q^4}{(m^2-q^2)^2}+\frac{5q^2}{2(m^2-q^2)}\right)\frac{\hat{s}}{\lambda}+(24m^2q^2+6q^4)\frac{\hat{s}}{\lambda^2}
$$

$$-\frac{8}{(\hat{s}-m^2)}+\left(8-\frac{q^4}{(m^2+q^2)^2}+\frac{3q^2}{2(m^2+q^2)}\right)\frac{1}{(\hat{s}-m^2-q^2)}\Bigg]. \qquad \text{(B.19)}$$

References

[1] W.L. van Neerven, Nucl. Phys. Proc.Suppl. 79 (1999) 36, hep-ph/9905503.

[2] J.D. Bjorken, Phys. Rev. **148** (1966) 1467, Phys. Rev. **D1** (1970) 1376.

[3] J.D. Bjorken, Phys. Rev. **163** (1967) 1767.

[4] D.J. Gross and C.H. Llewellyn Smith, Nucl. Phys. **B283** (1969) 337.

[5] S.A. Larin, F.V. Tkachov, J.A.M. Vermaseren, Phys. Rev. Lett. **66** (1991) 862.

[6] S.A. Larin and J.A.M. Vermaseren, Phys. Lett. **B259** (1991) 345.

[7] S. G. Gorishni, A.L. Kataev, S.A. Larin, Phys. Lett. **B159** (1991) 144;
L.R. Surguladze and M.A. Samuel, Phys. Rev. Lett. **66** (1991) 560, Erratum Phys. Rev. Lett. **66** (1991) 2416.

[8] S.A. Larin, T. van Ritbergen, J.A.M. Vermaseren,Phys. Lett. **B320** (1994) 159, ibid. Nucl. Phys. **B438** (1995) 278;
K. G. Chetyrkin and O.V. Tarasov, Phys. Lett. **B327** (1994) 114.

[9] A. Wagner, Nucl. Phys. Proc. Suppl. 79 (1999) 643.

[10] "Physics with Polarized Protons at HERA", eds. A. de Roeck and T. Gehrmann. DESY-97-233, hep-ph/9711512;
"Polarized Protons at High Energies-Accelerator Challenges and Physics Opportunities", eds. A. de Roeck, D. Barber and G. Rädel, DESY-PROC-1999-03 ;
S.D. Bass and A. de Roeck, hep-ph/0008289.

[11] M. Anselmino et al., "Electron Scattering with Polarized Targets at TESLA", hep-ph/0011299.

[12] S.L. Adler and R.F. Dashen, "Current Algebras and Applications to Particle Physics", in Frontiers in Physics, W.A. Benjamin Inc., New York, 1968.

[13] C.H. Llewellyn Smith, Phys. Rep. **3C** (1972) 261.

[14] J. Blümlein and N. Kochelev, Phys. Lett. **B381** (1996) 296.

[15] J. Blümlein and N. Kochelev, Nucl. Phys. **B498** (1997) 285.

[16] M. Gell-Mann, Physics **1**, (1964) 63.

[17] H. Fritzsch and M. Gell-Mann, Proceedings of the Coral Gables Conference on Fundamental Interactions at High Energies, January 1971, in "Scale Invariance and the Light Cone", Gordon and Breach (1971).

[18] J.M. Cornwall and R. Jackiw, Phys. Rev. **D4** (1971) 367.

[19] S.L. Adler, Phys. Rev. **143** (1966) 1144.

[20] J. Ellis and R. Jaffe, Phys. Rev. **D9** (1974) 1444, Erratum **D10** (1974) 1669.

[21] S.L. Adler, Phys. Rev. **177** (1969) 2426; J.S. Bell and R. Jackiw, Nuovo Cim. **A60** (1969) 47.

[22] K. Gottfried, Phys. Rev. Lett. **18** (1967) 1174.

[23] H. Burkhardt and W.N. Cottingham, Ann. Phys. (N.Y.) **56** (1970) 453.

[24] J. Blümlein and A. Tkabladze, Nucl. Phys. **B553** (1999) 427.

[25] X. Ji, Nucl. Phys. **B402** (1999) 217.

[26] R. Jackiw, "Dilatation Symmetry and Light-Cone Commutators", Lectures given at the University of Turin and the DESY Summer School, July 1971.

[27] C.G. Callan and D.J. Gross, Phys. Rev. Lett. **22** (1966) 156.

[28] D.A. Dicus, Phys. Rev. **D5** (1972) 1367.

[29] J. Ashman et al. (EMC), Phys. Lett. **B206** (1988) 364, ibid. Nucl. Phys. **B328** (1989) 1;
V.W. Hughes et al. Phys. Lett. **B212** (1988) 511.

[30] D.A. Ross, C.T Sachrajda, Nucl. Phys. **B149** (1979) 497.

[31] P. Amaudruz et al. (NMC), Phys. Rev. Lett. **66** (1991) 2712.

[32] J. Blümlein and W.L. van Neerven, Phys.Lett. **B450** (1999) 417;
W.L. van Neerven, hep-ph/9910356.

[33] B. Humpert and W.L. van Neerven, Nucl. Phys. **B184** (1981) 225.

[34] L. Lewin, "Polylogarithms and Associated Functions", North Holland, Amsterdam, 1983.

[35] O.V. Teryaev, preprint hep-ph/9508374;
O.V. Teryaev and O.L. Veretin, preprint hep-ph/9602362.

[36] E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. **B383** (1992) 525.

[37] E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. **B417** (1994) 61, Erratum **B426** (1994) 245, **B501** (1997) 599.

[38] I. Akushevich, A. Ilyichev, N. Shumeiko, hep-ph/9910475.

[39] W-Y Tsai, L.L. DeRaad and K.A. Milton, Phys. Rev. **D11** (1975) 3537.

[40] R. Mertig and W.L. van Neerven, Z. Phys. **C60** (1993) 489, Erratum ibid. **C65** (1995) 360.

[41] G. Altarelli, B. Lampe, P. Nason, G. Ridolfi, Phys. Lett. **B334** (1994) 187.